

# **AS and A Level Mathematics and Further Mathematics**

Consultation on Conditions and Guidance



December 2015

Ofqual/15/5792

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## **1. Introduction**

- 1.1 As most readers will know, changes are being made to GCSEs, AS and A levels taken by students in England. New AS and A level qualifications in mathematics and further mathematics will be taught in schools from September 2017.
- 1.2 The Department for Education (DfE) worked with the A Level Content Advisory Board (ALCAB) to develop the subject content for AS and A level mathematics and further mathematics. DfE published the subject content<sup>1</sup> in December 2014.
- 1.3 Following our own consultation on assessment arrangements<sup>2</sup> for these subjects, we confirmed<sup>3</sup> in December 2014 that new AS and A levels in mathematics and further mathematics will be assessed entirely through exams.

### **Scope of this consultation**

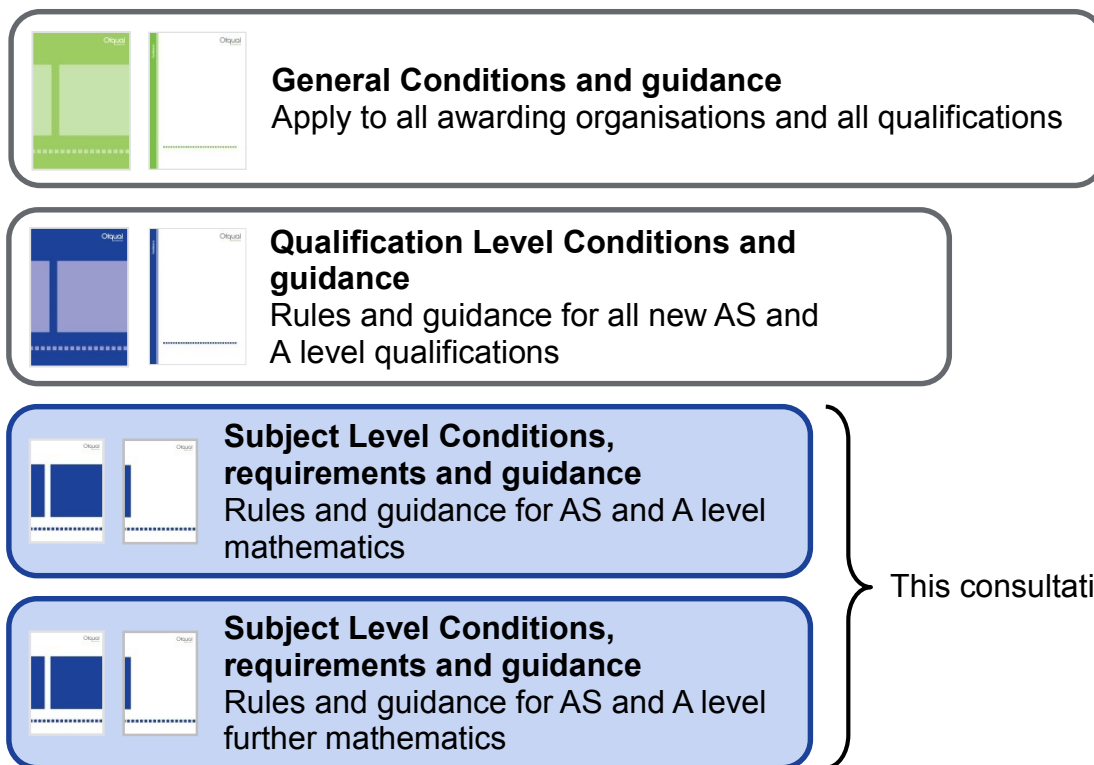
- 1.4 This consultation builds on our – and DfE’s – earlier decisions. It seeks views on the subject-specific rules and guidance we should put in place for AS and A level mathematics and further mathematics.

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<sup>1</sup> [www.gov.uk/government/publications/gce-as-and-a-level-mathematics](http://www.gov.uk/government/publications/gce-as-and-a-level-mathematics) and [www.gov.uk/government/publications/gce-as-and-a-level-further-mathematics](http://www.gov.uk/government/publications/gce-as-and-a-level-further-mathematics)

<sup>2</sup> [www.gov.uk/government/consultations/gcses-as-and-a-levels-reform-of-subjects-for-september-2016](http://www.gov.uk/government/consultations/gcses-as-and-a-levels-reform-of-subjects-for-september-2016)

<sup>3</sup> [www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/389473/confirmed-assessment-arrangements-for-reformed-gcses-as-qualifications-and-a-levels.pdf](http://www.gov.uk/government/uploads/system/uploads/attachment_data/file/389473/confirmed-assessment-arrangements-for-reformed-gcses-as-qualifications-and-a-levels.pdf)



1.5 As explained in Appendix A, these new rules and guidance will sit alongside our existing rules and guidance for

- all qualifications,<sup>4</sup> and
- all new AS and A level qualifications.<sup>5</sup>

1.6 Our decisions on the rules and guidance we need to put in place have been informed by advice from the A Level Mathematics Working Group – a panel including representatives of ALCAB, subject associations, teachers and exam boards. We set up the Working Group in March 2015 to provide expert advice on mathematical problem-solving, mathematical modelling and the use of large data sets. We are publishing the Working Group’s final report and advice alongside this consultation.

1.7 This document sets out, and seeks views on:

- a revised version of our assessment objectives;

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<sup>4</sup> [www.gov.uk/guidance/awarding-organisations-understanding-our-regulatory-requirements#requirements-for-all-awarding-organisations-and-all-regulated-qualifications](http://www.gov.uk/guidance/awarding-organisations-understanding-our-regulatory-requirements#requirements-for-all-awarding-organisations-and-all-regulated-qualifications)

<sup>5</sup> [www.gov.uk/government/collections/new-a-level-and-as-level-qualifications-requirements-and-guidance](http://www.gov.uk/government/collections/new-a-level-and-as-level-qualifications-requirements-and-guidance)

- our proposed approach to regulating new AS and A level qualifications in mathematics and further mathematics; and
- the subject-specific Conditions, requirements and guidance we propose to introduce to implement that approach.

1.8 This document also includes two proposed new appendices to the subject content – covering mathematical notation and formulae which students will be expected to know. Since these appendices relate to the subject content, they have been produced by DfE, in conjunction with ALCAB. We are consulting on these on behalf of DfE; they have been included in this consultation so that stakeholders can see all the proposals in one place, and only need to respond to a single consultation. We will pass responses on the proposed new appendices to the DfE which will then decide whether or not to include them in the subject content. If the DfE does decide to include the new appendices they will be covered by our proposed requirement for exam boards to comply with the subject content.

## How to respond to this consultation

The closing date for responses is 11th January 2016.

Please respond to this consultation in one of three ways:

- Complete the online response at [www.surveygizmo.com/s3/2461563/as-and-a-level-mathematics-and-further-mathematics-conditions-and-guidance](http://www.surveygizmo.com/s3/2461563/as-and-a-level-mathematics-and-further-mathematics-conditions-and-guidance).
- Email your response to [consultations@ofqual.gov.uk](mailto:consultations@ofqual.gov.uk) – please include the consultation title (AS and A Level Maths Consultation 2015) in the subject line of the email and make clear who you are and in what capacity you are responding; or
- Post your response to: AS and A Level Maths Consultation 2015, Ofqual, Spring Place, Coventry Business Park, Herald Avenue, Coventry, CV5 6UB, making clear who you are and in what capacity you are responding.

### Evaluating the responses

To evaluate responses properly, we need to know who is responding to the consultation and in what capacity. We will therefore only consider your response if you complete the information page.

Any personal data (such as your name, address and any other identifying information) will be processed in accordance with the Data Protection Act 1998 and our standard terms and conditions.

We will publish our evaluation of responses. Please note that we may publish all or part of your response unless you tell us (in your answer to the confidentiality question) that you want us to treat your response as confidential. If you tell us you wish your response to be treated as confidential, we will not include your details in any published list of respondents, although we may quote from your response anonymously.

Please respond by 11th January 2016.

## 2. Assessment objectives

- 2.1 Our earlier consultation on assessment arrangements<sup>6</sup> for AS and A level mathematics and further mathematics included draft assessment objectives.
- 2.2 The skills students need to demonstrate are the same in both mathematics and further mathematics – the difference between the subjects is that they require students to use those skills with different content. As a result, we initially proposed to introduce the same assessment objectives for both mathematics and further mathematics.
- 2.3 As a result both of responses to that consultation, and of input from the A Level Mathematics Working Group, we have made some changes both to the wording and weighting of our proposed assessment objectives. We are therefore giving all stakeholders a further opportunity to comment on them. The proposed assessment objectives are, again, the same for both mathematics and further mathematics.

### Assessment objective wording

- 2.4 We have made a number of changes to the wording of the assessment objectives from those previously proposed<sup>7</sup> – in particular, we have:
- removed ‘use and interpret notation correctly’ from AO1 – this is now included within ‘use mathematical language correctly’ in AO2;
  - within AO2, made explicit that students should be able to explain their reasoning;
  - clarified the wording of all three assessment objectives.

	Objective
<b>AO1</b>	<b>Use and apply standard techniques</b> Learners should be able to: <ul style="list-style-type: none"><li>■ select and correctly carry out routine procedures; and</li><li>■ accurately recall facts, terminology and definitions.</li></ul>
<b>AO2</b>	<b>Reason, interpret and communicate mathematically</b> Learners should be able to: <ul style="list-style-type: none"><li>■ construct rigorous mathematical arguments (including proofs);</li></ul>

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<sup>6</sup> [www.gov.uk/government/consultations/gcses-as-and-a-levels-reform-of-subjects-for-september-2016](http://www.gov.uk/government/consultations/gcses-as-and-a-levels-reform-of-subjects-for-september-2016)

<sup>7</sup> The assessment objectives that we previously consulted on in our *Developing new GCSE, A level and AS qualifications for first teaching in 2016* consultation have been included at Appendix B to this consultation.



	<ul style="list-style-type: none"><li>■ make deductions and inferences;</li><li>■ assess the validity of mathematical arguments;</li><li>■ explain their reasoning; and</li><li>■ use mathematical language correctly.</li></ul> <p><i>Where questions/tasks targeting this assessment objective will also credit Learners for the ability to ‘use and apply standard techniques’ (AO1) and/or to ‘solve problems within mathematics and other contexts’ (AO3) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).</i></p>
<b>AO3</b>	<p><b>Solve problems within mathematics and in other contexts</b></p> <p>Learners should be able to:</p> <ul style="list-style-type: none"><li>■ translate problems in mathematical and non-mathematical contexts into mathematical processes;</li><li>■ interpret solutions in the context of a problem, and, where appropriate, evaluate their accuracy and limitations;</li><li>■ translate situations in context into mathematical models;</li><li>■ use mathematical models; and</li><li>■ evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them.</li></ul> <p><i>Where questions/tasks targeting this assessment objective will also credit Learners for the ability to ‘use and apply standard techniques’ (AO1) and/or to ‘reason, interpret and communicate mathematically’ (AO2) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).</i></p>

**Question 1: To what extent do you agree or disagree that the proposed assessment objectives are appropriate for AS and A level mathematics and further mathematics?**

## Weighting of assessment objectives

- 2.5 We now believe we also need to take a different approach to the weighting of the assessment objectives.
- 2.6 First, we think we need to increase the weighting of AO1 – this is because even the most complex questions targeting AO2 and AO3 will very likely require use of standard techniques. There needs to be sufficient weighting assigned to AO1 to enable appropriate reward to be given for these skills.
- 2.7 We also think we need to specify different assessment objective weightings for mathematics and further mathematics. This is because the subject content in mathematics specifies all the topics exam boards must include in their

specifications, but the subject content in further mathematics specifies the core content then allows exam boards to choose a significant proportion of additional topics (accounting for 50 per cent of the specification at A level, and 70 per cent at AS). Our view is that the most appropriate weighting of the assessment objectives for further mathematics will therefore depend on exam boards' choice of topics.

- 2.8 Consequently, in mathematics we propose to set fixed weightings for all three assessment objectives, but – in further mathematics – we propose to set a fixed weighting for AO1, and minimum weightings for AO2 and AO3.
- 2.9 When setting fixed weightings for assessment objectives, there is a risk that assessment design will be constrained in unhelpful ways. In extreme cases, an exam board might be unable to use their preferred questions because they would not fit with the prescribed weightings.
- 2.10 We do not want the way we regulate AS and A level mathematics and further mathematics to constrain assessment design in this way. To avoid that, we proposed to allow exam boards a limited tolerance ( $\pm 2$  per cent) for all the fixed weightings.
- 2.11 For example, we have specified a weighting of 50 per cent for AO1 in A level mathematics – allowing a 2 per cent tolerance means the weighting of AO1 in a particular specification and year could vary between 48 per cent and 52 per cent (with the weightings of AO2 and AO3 adjusted accordingly).
- 2.12 Overall, this means we are proposing the following weightings for the assessment objectives in AS and A level mathematics and further mathematics:

Objective	Mathematics		Further mathematics	
	A level	AS	A level	AS
AO1 – Use and apply standard techniques	50% ( $\pm 2\%$ )	60% ( $\pm 2\%$ )	50% ( $\pm 2\%$ )	60% ( $\pm 2\%$ )
AO2 – Reason, interpret and communicate mathematically	25% ( $\pm 2\%$ )	20% ( $\pm 2\%$ )	At least 15%	At least 10%
AO3 – Solve problems within mathematics and in other contexts	25% ( $\pm 2\%$ )	20% ( $\pm 2\%$ )	At least 15%	At least 10%

- 2.13 We think this will give exam boards enough flexibility to design their assessments in the best way, but without compromising the overall level of demand of the qualification or the curriculum intentions.

**Question 2: To what extent do you agree or disagree that the proposed weightings of the assessment objectives are appropriate for AS mathematics?**

**Question 3: To what extent do you agree or disagree that the proposed weightings of the assessment objectives are appropriate for A level mathematics?**

**Question 4: To what extent do you agree or disagree that the proposed weightings of the assessment objectives are appropriate for AS further mathematics?**

**Question 5: To what extent do you agree or disagree that the proposed weightings of the assessment objectives are appropriate for A level further mathematics?**

### **3. Regulating AS and A level mathematics and further mathematics**

- 3.1 Mathematics and further mathematics are closely related subjects, and we propose to take a similar approach to regulating them. At the same time, there are differences in the subject content that mean some aspects of the way we regulate need to be different for mathematics and further mathematics.
- 3.2 This consultation therefore sets out separately the rules and guidance which we are proposing should apply to:
- both mathematics and further mathematics;
  - mathematics only; and
  - further mathematics only.

#### **Rules and guidance for both mathematics and further mathematics**

##### **Compliance with subject content and assessment objectives**

- 3.3 As we explained in paragraph 1.2, DfE has published the subject content for new AS and A levels in mathematics<sup>8</sup> and further mathematics.<sup>9</sup> (As noted in paragraph 2.7, this comprises all the required content for mathematics and the required core content for further mathematics.)
- 3.4 One of the ways we ensure new AS and A levels are comparable is by requiring them to be in line with the relevant subject content and our assessment objectives.
- 3.5 The approach we have taken in every other new AS and A level qualification is to introduce subject-specific Conditions which:
- require exam boards to comply with the requirements of the subject content (and have regard to any guidance that we publish in relation to the subject content); and
  - require exam boards to comply with our assessment objectives (and have regard to our guidance on those assessment objectives).

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<sup>8</sup> [www.gov.uk/government/publications/gce-as-and-a-level-mathematics](http://www.gov.uk/government/publications/gce-as-and-a-level-mathematics)

<sup>9</sup> [www.gov.uk/government/publications/gce-as-and-a-level-further-mathematics](http://www.gov.uk/government/publications/gce-as-and-a-level-further-mathematics)

- 3.6 Although we do not always specify how the subject content should be interpreted, we think it is important for us to be able to do so when there is a good reason for that (for example, if a different interpretation could compromise qualification standards or comparability). And – as set out in more detail below – we think we do need to specify how some parts of the mathematics and further mathematics subject content should be interpreted.
- 3.7 We see no reason to take a different approach for either mathematics or further mathematics. We are therefore proposing that, for both mathematics and further mathematics, we should introduce a Condition which requires exam boards to:
- comply with the requirements of the subject content – this includes requirements in the proposed new appendices;
  - comply with any requirements (and have regard to any guidance) we publish on interpreting the subject content; and
  - comply with our assessment objectives (and have regard to our guidance on those assessment objectives).

**Question 6: To what extent do you agree or disagree that we should introduce a Condition which requires exam boards to comply with the relevant subject content – including the two proposed new appendices – and assessment objectives?**

### **Interpreting the subject content**

- 3.8 The subject content for both mathematics and further mathematics includes the following two sections:
- ‘Overarching themes’, which sets out knowledge and skills that students are expected to demonstrate across the detailed content statements; and
  - ‘Use of technology’, which explains how technology should be used in the study of mathematics and further mathematics.
- 3.9 Both these sections largely set expectations about the way in which AS and A level mathematics and further mathematics should be taught in schools. But it is also important that assessments reflect them.
- 3.10 We believe we need to clarify that these sections should be reflected in the design of AS and A level mathematics and further mathematics specifications. We are therefore proposing to introduce guidance that clarifies that we expect awarding organisations to explain and justify in their assessment strategies how their qualification design reflects these sections of the subject content.

**Question 7: To what extent do you agree or disagree that we should introduce guidance which clarifies that awarding organisations should explain and justify in their assessment strategies how their qualification design reflects the ‘Overarching themes’ and ‘Use of technology’ sections of the subject content?**

### **Use of calculators**

- 3.11 For new GCSEs in mathematics, we have previously specified rules which limit the functions of calculators students can use in assessments. We had originally intended to introduce similar rules for new AS and A level mathematics and further mathematics.
- 3.12 However, concerns about the suitability of this approach for AS and A level have prompted us to reconsider our position. We are consulting separately<sup>10</sup> on how we should regulate the use of calculators in assessments for all new GCSE, AS and A level qualifications, including for new AS and A level mathematics and further mathematics.

### **Guidance on assessment objectives**

- 3.13 For all other new AS and A level qualifications, we have published guidance which explains how awarding organisations should interpret our assessment objectives. This is designed to ensure awarding organisations have a common understanding of – and take a consistent approach to targeting – the different assessment objectives.
- 3.14 We are proposing we should introduce similar guidance for mathematics and further mathematics.
- 3.15 Our proposed guidance on assessment objectives incorporates – where appropriate – the A Level Mathematics Working Group’s advice on mathematical problem-solving and mathematical modelling.

**Question 8: To what extent do you agree or disagree that we should introduce guidance which clarifies how awarding organisations should interpret our assessment objectives?**

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<sup>10</sup> [www.gov.uk/government/consultations/regulating-use-of-calculators-in-new-gcses-as-and-a-levels](http://www.gov.uk/government/consultations/regulating-use-of-calculators-in-new-gcses-as-and-a-levels)

## **Rules and guidance for mathematics only**

### **Date of first award for A level**

3.16 Mathematics is the only A level subject area where there are two A levels, one of which builds on the other. As a result of this, students taking A levels in both mathematics and further mathematics can choose to take their exams in a number of different ways. Students typically do one of the following:

- take both mathematics and further mathematics A levels at the end of year 13; or
- take mathematics A level at the end of year 12, followed by further mathematics A level at the end of year 13.

3.17 The way we have introduced new A levels in all other subjects only allows the first exams to take place two years after the subject is first introduced. Allowing both the legacy A level and the new A level to be taken by the same year group would make it significantly more difficult for exam boards to set standards. In turn, this could reduce confidence in the level at which grades had been awarded in a year where both A levels were available. In addition, the simultaneous availability of exams in two subjects of the same titles with different assessment arrangements has the potential to be confusing for schools and colleges.

3.18 If we took the same approach here, students who start studying the new A levels in September 2017 would not be allowed to take mathematics A level at the end of year 12, but this would be available to students in subsequent years.

3.19 This is the only A level subject where this is an issue, and the potential impact on students is significant. We have therefore considered whether there is a case for making an exception for A level mathematics, and whether allowing simultaneous availability of the legacy and new qualifications is proportionate in these specific circumstances.

3.20 We have sought further information from exam boards to understand the likely impact of allowing both the legacy and new mathematics A levels to be taken in 2018. Because – uniquely among A level subjects – a significant number of students currently take mathematics A level at the end of year 12, we think that the potential impacts on setting standards can be managed sufficiently in mathematics. Indeed, given the patterns of entry in these subjects, not allowing students who would usually sit the mathematics A level at the end of year 12 to do so would pose a challenge to standard setting in summer 2019, since this would be an atypical group, different to any that would be likely to be present in subsequent years.

- 3.21 Taking all this into account, our view is that we should make an exception for new A levels in mathematics, and that the first opportunity for students to sit their exams in this qualification should be in summer 2018.

**Question 9: To what extent do you agree or disagree that we should allow the first exams for new mathematics A levels in summer 2018 (at the end of the first year of teaching)?**

### **Sampling of subject content**

- 3.22 The subject content for AS and A level mathematics includes a series of detailed content statements. These statements must comprise 100 per cent of the qualification at both AS and A level.

- 3.23 While this means exam boards' specifications must contain the same content, it does not necessarily mean they will take a consistent approach to assessing it. In principle, an exam board could choose to focus assessments on particular areas of the content, and only assess other areas infrequently. Such an approach could have an impact on teaching, as it could encourage a narrower focus on the topics which most frequently feature in assessments. It could also have an adverse effect on comparability over time if the range of content assessed varied markedly year to year.

- 3.24 To prevent this, we believe we need to set subject-specific rules which require exam boards to cover in their assessments all the subject content in as short a period of time as possible, and cover an appropriate range of subject content in each year's exams. In doing so, exam boards will also need to ensure they minimise the predictability of assessments, and promote comparability of assessments over time.

**Question 10: To what extent do you agree or disagree with our proposed approach to regulating the sampling of subject content in AS and A level mathematics?**

### **Large data sets**

- 3.25 The subject content for mathematics includes a section headed 'Use of data in statistics'. This section sets clear expectations that exam boards will design their specifications to include one or more large data sets which schools will be expected to use when teaching statistics, and that assessments should reflect this.



- 3.26 Assessments should support the curriculum intention – in this case a desire to change the way statistics is taught, so that students use and manipulate large data sets in the classroom.
- 3.27 The use of large data sets was one of the issues considered by our A level Mathematics Working Group. The Working Group identified a number of important characteristics that would make a data set suitable for use by schools, as well as different ways in which questions could be designed to reflect this approach to teaching.
- 3.28 Assessment of large data sets in examinations is challenging, because there are inherent limitations on the time and resources (such as technology) which are available to students. With these constraints on what is manageable in mind, our view is that the emphasis in assessments should therefore be on assessing familiarity with the specified large data sets.
- 3.29 Building on the Working Group’s advice, our view is that questions in exams which target large data sets should be designed to give a material advantage to students who have studied, and are therefore familiar with, the specific large data set(s) provided by the exam board. We propose to introduce guidance which clarifies this expectation, highlights the characteristics of questions which might achieve it, and sets out the characteristics of good data sets.

**Question 11: To what extent do you agree or disagree with our proposed approach to regulating the use and assessment of large data sets in AS and A level mathematics?**

## **Rules and guidance for further mathematics only**

### **Non-core subject content**

- 3.30 The subject content for new AS and A level further mathematics sets out the ‘core content’ in a series of detailed content statements. Unlike in mathematics, this core content makes up 50 per cent of the qualification at A level, and 30 per cent at AS. Exam boards must then choose additional topics for the rest of the qualification.
- 3.31 This intentionally gives exam boards freedom to develop different approaches to further mathematics which focus on different specialisms within mathematics. For example, one exam board might choose to focus more on pure mathematics, and another on mechanics or statistics. Exam boards can also choose to offer different options within a single specification.
- 3.32 We do not think exam boards should have a completely free choice when selecting the additional content to include in their specifications. We think any

additional content should reflect the needs of qualification users (for example, employers and higher education institutions), and that there should be an appropriate amount of additional content. We also think that the weightings the exam boards propose for AO2 and AO3 should reflect the overall content of the specification (both core and non-core).

3.33 Exam boards also need to ensure that assessments are comparable over time. And if there are any optional routes through the qualification, exam boards will need to ensure those routes are comparable (in line with General Condition G1.2, which requires optional routes through qualifications to be at the same level of demand).

3.34 We therefore propose to set subject-specific rules which require exam boards to explain and justify their approach to non-core content in their assessment strategies.

3.35 We also propose to publish guidance which highlights the information we expect exam boards to provide when explaining how they have secured comparability between any optional routes. We have already introduced similar guidance for new GCSE, AS and A level religious studies, where it is possible to have a large number of optional routes, and we believe something similar would be helpful here.

**Question 12: To what extent do you agree or disagree with our proposed approach to regulating non-core content in AS and A level further mathematics?**

## Our proposed Conditions and guidance

4.1 As set out above, we are proposing to introduce subject-specific Conditions, requirements and guidance to implement the proposals in this consultation. We set out our proposed Conditions, requirements and guidance for each of mathematics and further mathematics below.

### Proposed Conditions and requirements for AS and A level mathematics

4.2 We are proposing to introduce the following Conditions and requirements which will apply to all new AS and A level qualifications in mathematics:

- a Condition covering compliance with subject content and assessment objectives;
- a further enabling Condition which allows us to specify more detailed requirements on assessment;
- our assessment objectives;<sup>11</sup> and
- requirements on assessment which relate to sampling of subject content.

<b>Condition GCE(Mathematics)1</b>	<b>Compliance with content requirements</b>
GCE(Mathematics)1.1	In respect of each GCE Qualification in Mathematics which it makes available, or proposes to make available, an awarding organisation must – <ul style="list-style-type: none"> <li>(a) comply with the requirements relating to that qualification set out in the document published by the Secretary of State entitled ‘Mathematics AS and A level content’<sup>12</sup>, document reference DFE-00706-2014,</li> </ul>

<sup>11</sup> Aside from the weightings, the proposed assessment objectives are common across mathematics and further mathematics.

<sup>12</sup> [www.gov.uk/government/publications/gce-as-and-a-level-mathematics](http://www.gov.uk/government/publications/gce-as-and-a-level-mathematics) – please note that the current published version does not include the proposed appendices we are seeking views on in this consultation. If the DfE decides not to include those two new appendices in the content then the requirement in GCE(Mathematics)1.1 will require compliance with the current published version of the subject content. If the DfE decides to include those new appendices, then the condition will require compliance with the subject content as revised by the DfE following this consultation.

GCE(Mathematics)1.2	<p>(b) have regard to any recommendations or guidelines relating to that qualification set out in that document, and</p> <p>(c) interpret that document in accordance with any requirements, and having regard to any guidance, which may be published by Ofqual and revised from time to time.</p> <p>In respect of each GCE Qualification in Mathematics which it makes available, or proposes to make available, an awarding organisation must comply with any requirements, and have regard to any guidance, relating to the objectives to be met by any assessment for that qualification which may be published by Ofqual and revised from time to time.</p>
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**Condition**

**Assessment**

**GCE(Mathematics)2**

GCE(Mathematics)2.1

An awarding organisation must ensure that in respect of each assessment for a GCE Qualification in Mathematics which it makes available it complies with any requirements, and has regard to any guidance, which may be published by Ofqual and revised from time to time.

**Assessment objectives – GCE Qualifications in Mathematics**

Condition GCE(Mathematics)1.2 allows us to specify requirements relating to the objectives to be met by any assessment for GCE Qualifications in Mathematics.

The assessment objectives set out below constitute requirements for the purposes of Condition GCE(Mathematics)1.2. Awarding organisations must comply with these requirements in relation to all GCE AS and A level Qualifications in Mathematics they make available.

	Objective	Weighting	
		A level	AS
<b>AO1</b>	<p><b>Use and apply standard techniques</b></p> <p>Learners should be able to:</p> <ul style="list-style-type: none"> <li>■ select and correctly carry out routine procedures; and</li> </ul>	50% (±2%)	60% (±2%)

	<ul style="list-style-type: none"> <li>■ accurately recall facts, terminology and definitions.</li> </ul>		
<b>AO2</b>	<p><b>Reason, interpret and communicate mathematically</b></p> <p>Learners should be able to:</p> <ul style="list-style-type: none"> <li>■ construct rigorous mathematical arguments (including proofs);</li> <li>■ make deductions and inferences;</li> <li>■ assess the validity of mathematical arguments;</li> <li>■ explain their reasoning; and</li> <li>■ use mathematical language correctly.</li> </ul> <p><i>Where questions/tasks targeting this assessment objective will also credit Learners for the ability to ‘use and apply standard techniques’ (AO1) and/or to ‘solve problems within mathematics and other contexts’ (AO3) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).</i></p>	25% (±2%)	20% (±2%)
<b>AO3</b>	<p><b>Solve problems within mathematics and in other contexts</b></p> <p>Learners should be able to:</p> <ul style="list-style-type: none"> <li>■ translate problems in mathematical and non-mathematical contexts into mathematical processes;</li> <li>■ interpret solutions in the context of a problem, and, where appropriate, evaluate their accuracy and limitations;</li> <li>■ translate situations in context into mathematical models;</li> <li>■ use mathematical models; and</li> <li>■ evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them.</li> </ul> <p><i>Where questions/tasks targeting this assessment objective will also credit Learners for the ability to ‘use and apply standard techniques’ (AO1) and/or to ‘reason, interpret and communicate mathematically’ (AO2) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).</i></p>	25% (±2%)	20% (±2%)

In respect of each GCE Qualification in Mathematics which it makes available, or proposes to make available, an awarding organisation must design and set the assessments on the basis that the weightings in respect of each of the assessment objectives outlined above may vary by  $\pm 2\%$  in each set of assessments.<sup>13</sup>

### **Assessment requirements – GCE Qualifications in Mathematics**

Condition GCE(Mathematics)2.1 allows us to specify requirements and guidance in relation to assessments for GCE Qualifications in Mathematics.

We set out our requirements for the purposes of Condition GCE(Mathematics)2.1 below.

#### **Sampling of subject content**

The subject content for GCE Qualifications in Mathematics is set out in the Department for Education's *Mathematics AS and A level content*, document reference DFE-00706-2014 (the 'Content Document').

In respect of each GCE Qualification in Mathematics which it makes available, or proposes to make available, an awarding organisation must demonstrate to Ofqual's satisfaction in its assessment strategy for that qualification that it has designed and set the assessments in a way which –

- (a) covers all of the detailed content statements (set out on pages 7 to 16 of the Content Document) in the fewest sets of assessments as is reasonably practicable,
- (b) ensures appropriate coverage of the detailed content statements in each set of assessments,
- (c) minimises predictability of assessments, and
- (d) promotes comparability of assessments (including over time).

### **Proposed Conditions and requirements for AS and A level further mathematics**

4.3 We are proposing to introduce the following Conditions and requirements which will apply to all new AS and A level qualifications in further mathematics:

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<sup>13</sup> For the purposes of these requirements a 'set of assessments' means the assessments to be taken by a particular Learner for a GCE Qualification in Mathematics. For clarity, the assessments taken by Learners may vary, depending on any possible routes through the qualification.

- a Condition covering compliance with subject content and assessment objectives,
- requirements for interpreting subject content which relate to non-core content, and
- our assessment objectives.<sup>14</sup>

**Condition GCE(Further Compliance with content requirements Mathematics)1**

GCE(Further Mathematics)1.1

In respect of each GCE Qualification in Further Mathematics which it makes available, or proposes to make available, an awarding organisation must –

- (a) comply with the requirements relating to that qualification set out in the document published by the Secretary of State entitled ‘Further mathematics AS and A level content’,<sup>15</sup> document reference DFE-00707-2014,
- (b) have regard to any recommendations or guidelines relating to that qualification set out in that document, and
- (c) interpret that document in accordance with any requirements, and having regard to any guidance, which may be published by Ofqual and revised from time to time.

GCE(Further Mathematics)1.2

In respect of each GCE Qualification in Further Mathematics which it makes available, or proposes to make available, an awarding organisation must comply with any requirements, and have regard to any guidance, relating to the objectives to be met by any assessment for that qualification which may be published by Ofqual and revised from time to time.

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<sup>14</sup> Aside from the weightings, the proposed assessment objectives are common across mathematics and further mathematics.

<sup>15</sup> [www.gov.uk/government/publications/gce-as-and-a-level-further-mathematics](http://www.gov.uk/government/publications/gce-as-and-a-level-further-mathematics) – please note that the current published version does not include the proposed appendices we are seeking views on in this consultation. If the DfE decides not to include those two new appendices in the content then the requirement in GCE(Further Mathematics)1.1 will require compliance with the current published version of the subject content. If the DfE decides to include those new appendices, then the condition will require compliance with the subject content as revised by the DfE following this consultation.

## Interpretation of subject content – GCE Qualifications in Further Mathematics

The subject content for GCE Qualifications in Further Mathematics is set out in the Department for Education's *Further Mathematics AS and A level content*, document reference DFE-00707-2014 (the 'Content Document').

Condition GCE(Further Mathematics)1.1(c) requires awarding organisations to interpret the Content Document in line with any requirements published by Ofqual.

We set out our requirements for the purposes of Condition GCE(Further Mathematics)1.1(c) below.

### Non-core subject content

Paragraphs 6 and 8 of the Content Document state that –

6. A level further mathematics has a prescribed core which must comprise approximately 50% of its content.

[...]

For the remaining 50% of the content, different options are available. The content of these options is not prescribed and will be defined within the different awarding organisations' specifications; these options could build from the applied content in A level mathematics, they could introduce new applications, or they could extend further the core content defined below, or they could involve some combination of these. Any optional content must be at the same level of demand as the prescribed core.

[...]

8. At least 30% (approximately) of the content of any AS further mathematics specification must be taken from the prescribed core content of A level further mathematics.

The Content Document goes on to set out the core content for GCE Qualifications in Further Mathematics.

In respect of each GCE Qualification in Further Mathematics which it makes available, or proposes to make available, an awarding organisation must explain and justify in its assessment strategy for the qualification –

- (a) the non-core content it has included within its specification, including, in particular –



- (i) how that non-core content reflects engagement with, and the needs of, Users of the qualification, and
- (ii) the range and amount of non-core content, including how this reflects the prescribed proportions of core and non-core content,
- (b) its proposed weightings for assessment objectives AO2 and AO3, including –
  - (i) how and why those weightings reflect the content (both core and non-core) in its specification,
  - (ii) any intended variation in those weightings over time, and
  - (iii) any intended variation in those weightings between any optional routes through the qualification,
- (c) the rationale for any optional routes through the qualification, and
- (d) how it will secure Comparability, including over time and between any optional routes through the qualification.

### Assessment objectives – GCE Qualifications in Further Mathematics

Condition GCE(Further Mathematics)1.2 allows us to specify requirements relating to the objectives to be met by any assessment for GCE Qualifications in Further Mathematics.

The assessment objectives set out below constitute requirements for the purposes of Condition GCE(Further Mathematics)1.2. Awarding organisations must comply with these requirements in relation to all GCE AS and A level Qualifications in Further Mathematics they make available.

	Objective	Weighting	
		A level	AS
<b>AO1</b>	<b>Use and apply standard techniques</b> Learners should be able to: <ul style="list-style-type: none"> <li>■ select and correctly carry out routine procedures; and</li> <li>■ accurately recall facts, terminology and definitions.</li> </ul>	50% (±2%)	60% (±2%)
<b>AO2</b>	<b>Reason, interpret and communicate mathematically</b> Learners should be able to: <ul style="list-style-type: none"> <li>■ construct rigorous mathematical arguments (including proofs);</li> <li>■ make deductions and inferences;</li> </ul>	At least 15%	At least 10%

	<ul style="list-style-type: none"> <li>■ assess the validity of mathematical arguments;</li> <li>■ explain their reasoning; and</li> <li>■ use mathematical language correctly.</li> </ul> <p><i>Where questions/tasks targeting this assessment objective will also credit Learners for the ability to ‘use and apply standard techniques’ (AO1) and/or to ‘solve problems within mathematics and other contexts’ (AO3) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).</i></p>		
<b>AO3</b>	<p><b>Solve problems within mathematics and in other contexts</b></p> <p>Learners should be able to:</p> <ul style="list-style-type: none"> <li>■ translate problems in mathematical and non-mathematical contexts into mathematical processes;</li> <li>■ interpret solutions in the context of a problem, and, where appropriate, evaluate their accuracy and limitations;</li> <li>■ translate situations in context into mathematical models;</li> <li>■ use mathematical models; and</li> <li>■ evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them.</li> </ul> <p><i>Where questions/tasks targeting this assessment objective will also credit Learners for the ability to ‘use and apply standard techniques’ (AO1) and/or to ‘reason, interpret and communicate mathematically’ (AO2) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).</i></p>	At least 15%	At least 10%

In respect of each GCE Qualification in Further Mathematics which it makes available, or proposes to make available, an awarding organisation must design and set the assessments on the basis that the weightings in respect of assessment objective AO1 outlined above may vary by  $\pm 2\%$  in each set of assessments.<sup>16</sup>

<sup>16</sup> For the purposes of these requirements a ‘set of assessments’ means the assessments to be taken by a particular Learner for a GCE Qualification in Further Mathematics. For clarity, the assessments taken by Learners may vary, depending on any possible routes through the qualification.

## Proposed guidance for AS and A level mathematics

4.4 We are proposing to introduce the following guidance which will apply to all new AS and A level qualifications in mathematics:

- guidance on interpretation of the subject content; and
- guidance on assessment objectives.<sup>17</sup>

4.5 Our proposed guidance on assessment objectives has been drafted based on the proposed assessment objectives set out above. If we make changes to the assessment objectives following this consultation, we may need to update the guidance to reflect these changes.

### **Guidance in relation to subject content for GCE Qualifications in Mathematics**

The subject content for GCE Qualifications in Mathematics is set out in the Department for Education's *Mathematics AS and A level content*, document reference DFE-00706-2014 (the 'Content Document').

Condition GCE(Mathematics)1.1(c) requires awarding organisations to interpret the Content Document in line with any requirements, and to have regard to any guidance, published by Ofqual.

We set out our guidance for the purposes of Condition GCE(Mathematics)1.1(c) below.

#### **Overarching themes and use of technology**

Paragraphs 7 and 8 of the Content Document state that –

7. A level specifications in mathematics must require students to demonstrate the following overarching knowledge and skills. These must be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content set out below.

*[list of overarching themes]*

8. The use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the study of AS and A level mathematics.

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<sup>17</sup> Aside from the weightings of the assessment objectives, the proposed guidance on assessment objectives are common across mathematics and further mathematics.

These statements should be interpreted primarily as indicating the desired approach to teaching GCE Qualifications in Mathematics.

However, these statements also have implications for assessments. Consequently, in respect of each GCE Qualification in Mathematics which it makes available, or proposes to make available, we expect an awarding organisation to explain and justify in its assessment strategy for that qualification how these statements have been reflected in the qualification's design.

### **Large data sets**

Paragraphs 9 and 10 of the Content Document state that –

9. AS and A level mathematics specifications must require students to:
  - become familiar with one or more specific large data set(s) in advance of the final assessment (these data must be real and sufficiently rich to enable the concepts and skills of data presentation and interpretation in the specification to be explored)
  - use technology such as spreadsheets or specialist statistical packages to explore the data set(s)
  - interpret real data presented in summary or graphical form
  - use data to investigate questions arising in real contexts
10. Specifications should require students to explore the data set(s), and associated contexts, during their course of study to enable them to perform tasks that assume familiarity with the contexts, the main features of the data and the ways in which technology can help explore the data. Specifications should also require students to demonstrate the ability to analyse a subset or features of the data using a calculator with standard statistical functions, as detailed in paragraph 8.

Awarding organisations should interpret the term 'specific large data set(s)' (and related terms) as meaning one or more data sets which:

- consist of real data – the data may be reorganised into a standard format but should not be cleansed by the awarding organisation;
- wherever possible, include the source of the data (including URLs) so that Learners can understand how it was collected;

- may be accompanied by a short piece of text and/or a glossary to help Learners understand the data and associated terminology;
- make clear to Learners whether the whole data set is (essentially) a population, or a sample from a larger population;
- are large enough to manage the risk of predictable assessments, while also being manageable for Centres and capable of supporting the requirements set out in the Content Document;
- contain a mixture of categorical and numerical data; and
- are suitable for analysis using a spreadsheet and/or statistical data package.

### **Questions/tasks targeting large data sets**

The Content Document sets out a clear expectation that assessments will include questions/tasks which relate to the specific large data set(s) studied.

Our expectation is that these questions/tasks should be likely to give a material advantage to Learners who have studied, and are familiar with, the prescribed large data set(s). They might include questions/tasks which:

- assume familiarity with the terminology and contexts of the data, and do not explain them in a way which provides Learners who have not studied the prescribed data set(s) the same opportunities to access marks as Learners who have studied them;
- use summary statistics or selected data from the prescribed large data set(s) – these might be provided within the question/task, or as Stimulus Materials;
- are based on samples related to the contexts in the prescribed large data set(s), so that Learners' work with the prescribed large data sets will help them understand the background context; and/or
- require Learners to interpret data in ways which would be too demanding in an unfamiliar context.

Where an awarding organisation provides a short piece of text and/or a glossary to help Learners understand the data and associated terminology, questions/tasks should require greater engagement with the data than the text and/or glossary provides.

### **Guidance on assessment objectives for GCE Qualifications in Mathematics**

Condition GCE(Mathematics)1.2 allows us to specify requirements and guidance relating to assessment objectives for GCE Qualifications in Mathematics.

We published our requirements in relation to assessment objectives in *GCE Subject Level Conditions and Requirements for Mathematics*, and reproduce them in the table below.

	Objective	Weighting	
		A level	AS
AO1	<p><b>Use and apply standard techniques</b> Learners should be able to:</p> <ul style="list-style-type: none"> <li>■ select and correctly carry out routine procedures; and</li> <li>■ accurately recall facts, terminology and definitions.</li> </ul>	50% (±2%)	60% (±2%)
AO2	<p><b>Reason, interpret and communicate mathematically</b> Learners should be able to:</p> <ul style="list-style-type: none"> <li>■ construct rigorous mathematical arguments (including proofs);</li> <li>■ make deductions and inferences;</li> <li>■ assess the validity of mathematical arguments;</li> <li>■ explain their reasoning; and</li> <li>■ use mathematical language correctly.</li> </ul> <p><i>Where questions/tasks targeting this assessment objective will also credit Learners for the ability to ‘use and apply standard techniques’ (AO1) and/or to ‘solve problems within mathematics and other contexts’ (AO3) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).</i></p>	25% (±2%)	20% (±2%)
AO3	<p><b>Solve problems within mathematics and in other contexts</b> Learners should be able to:</p> <ul style="list-style-type: none"> <li>■ translate problems in mathematical and non-mathematical contexts into mathematical processes;</li> <li>■ interpret solutions in the context of a problem, and, where appropriate, evaluate their accuracy and limitations;</li> <li>■ translate situations in context into mathematical models;</li> <li>■ use mathematical models; and</li> <li>■ evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them.</li> </ul>	25% (±2%)	20% (±2%)

	<p><i>Where questions/tasks targeting this assessment objective will also credit Learners for the ability to ‘use and apply standard techniques’ (AO1) and/or to ‘reason, interpret and communicate mathematically’ (AO2) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).</i></p>		
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We set out below our guidance for the purposes of Condition GCE(Mathematics)1.2. This guidance explains how we expect awarding organisations to interpret these assessment objectives in terms of:

- the different ‘strands’ within each of the assessment objectives;
- the discrete ‘elements’ within each assessment objective and its strands which questions and tasks could target and/or seek to credit – our expectation is that each and every question/task should target or seek to credit at least one of these elements, and may target or seek to credit multiple elements across one or more assessment objectives;
- the coverage expectations, such as in relation to the different elements within each assessment objective and how those elements should be sampled over time; and
- the key areas of emphasis in each assessment objective and the particular meaning for the subject of any key terms and phrases used; defined terms are shown in bold text, followed by their definitions.

In line with the obligations set out in Condition GCE(Mathematics)1.2, we expect awarding organisations to be able to demonstrate how they have had regard to this guidance. For example, an awarding organisation could map how it has regard to the guidance as it:

- develops its sample assessment materials;
- delivers the qualification;
- develops and applies its approach to sampling the elements into which the assessment objectives are divided; and
- monitors the qualification to make sure it addresses all elements appropriately.

AO1: Use and apply standard techniques			50% (A level)
Learners should be able to:			60% (AS)
<ul style="list-style-type: none"> <li>▪ select and correctly carry out routine procedures</li> <li>▪ accurately recall facts, terminology and definitions</li> </ul>			
Strands	Elements	Coverage	Interpretation and definitions
1 – select and correctly carry out routine procedures	1a – select routine procedures	<ul style="list-style-type: none"> <li>▪ Full coverage in each set of assessments (but not every assessment).</li> <li>▪ No more than 10% of the marks for this assessment objective should be allocated solely to strand 2.</li> </ul>	<ul style="list-style-type: none"> <li>▪ <b>Routine procedures</b> includes multi-step as well as single-step processes. They should be familiar to the Learner (including, but not limited to, those stated in the specification) and there should be no significant background context given in the question/task that would have an impact on the Level of Demand.</li> <li>▪ <b>Select</b> involves the recognition of a single- or multi-step process necessary to carry out a routine procedure (for example, solving a quadratic equation or integrating a function) in cases where the question/task does not make the required process clear. It should not be confused with the more complex decision-making required in AO3.</li> <li>▪ Element 1a should normally be assessed in combination with element 1b.</li> <li>▪ Within strand 2, individual questions/tasks may target each of facts, terminology and definitions in isolation, or in any combination.</li> </ul>
	1b – correctly carry out routine procedures		
2 – accurately recall facts, terminology and definitions	This strand is a single element.		



AO2: Reason, interpret and communicate mathematically			25% (A level)
Learners should be able to: <ul style="list-style-type: none"> <li>▪ construct rigorous mathematical arguments (including proofs)</li> <li>▪ make deductions and inferences</li> <li>▪ assess the validity of mathematical arguments</li> <li>▪ explain their reasoning</li> <li>▪ use mathematical language correctly</li> </ul>			20% (AS)
Strands	Elements	Coverage	Interpretation and definitions
<b>1 – construct rigorous mathematical arguments (including proofs)</b>	This strand is a single element.	<ul style="list-style-type: none"> <li>▪ Full coverage in each set of assessments (but not every assessment).</li> <li>▪ Taken together, strands 1 and 2 should comprise at least 50% of the marks for this assessment objective.</li> </ul>	<ul style="list-style-type: none"> <li>▪ A <b>mathematical argument</b> leads from premises to a conclusion, through rigorous and clear reasoning. It:                             <ul style="list-style-type: none"> <li>□ may include one or both of calculations or algebraic manipulation, but is more than these alone,</li> <li>□ typically involves several steps, which are logical in nature and sequence, given the context,</li> <li>□ clearly identifies the principal results used where appropriate, and</li> <li>□ contains sufficient detail to allow the line of reasoning to be followed.</li> </ul> </li> <li>▪ <b>Deduction</b> means a process of reasoning from absolutes to results that must be correct.</li> <li>▪ <b>Inference</b> means a process of reasoning from relative or partial evidence to results that are likely to be correct.</li> <li>▪ Strand 3 could apply to arguments provided to a Learner, or to arguments generated by them. It may include identifying:                             <ul style="list-style-type: none"> <li>□ errors and/or omissions; and</li> </ul> </li> </ul>
<b>2 – make deductions and inferences</b>	2a – make deductions 2b – make inferences		
<b>3 – assess the validity of mathematical arguments</b>	This strand is a single element.	<ul style="list-style-type: none"> <li>▪ No more than 10% of the marks for this assessment objective should</li> </ul>	
<b>4 – explain their reasoning</b>	This strand is a single element.		

<b>AO2: Reason, interpret and communicate mathematically</b>			25% (A level)
Learners should be able to: <ul style="list-style-type: none"> <li>▪ construct rigorous mathematical arguments (including proofs)</li> <li>▪ make deductions and inferences</li> <li>▪ assess the validity of mathematical arguments</li> <li>▪ explain their reasoning</li> <li>▪ use mathematical language correctly</li> </ul>			20% (AS)
Strands	Elements	Coverage	Interpretation and definitions
<b>5 – use mathematical language correctly</b>	This strand is a single element.	be allocated to strand 3. <ul style="list-style-type: none"> <li>▪ No more than 10% of the marks for this assessment objective should be allocated solely to strand 5.</li> </ul>	<ul style="list-style-type: none"> <li>□ the values for, and/or conditions under, which an argument remains correct.</li> <li>▪ In the context of strand 4, <b>explain</b> may include providing justification for a particular approach. Justification and/or explanation of key steps in the working should be required even where problems are otherwise fairly routine in nature.</li> <li>▪ The correct use of mathematical notation is included within strand 5.</li> <li>▪ Strands 4 and 5 should normally be assessed in combination with strands 1, 2 and/or 3. Learners should be given multiple opportunities to demonstrate (and gain credit for) the skills required by these strands.</li> </ul>

<b>AO3: Solve problems within mathematics and in other contexts</b>			25% (A level)
Learners should be able to:			20% (AS)
<ul style="list-style-type: none"> <li>▪ translate problems in mathematical and non-mathematical contexts into mathematical processes</li> <li>▪ interpret solutions in the context of a problem, and, where appropriate, evaluate their accuracy and limitations</li> <li>▪ translate situations in context into mathematical models</li> <li>▪ use mathematical models</li> <li>▪ evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them</li> </ul>			
Strands	Elements	Coverage	Interpretation and definitions
<b>1 – translate problems in mathematical and non-mathematical contexts into mathematical processes</b>	1a – translate problems in mathematical contexts into mathematical processes	<ul style="list-style-type: none"> <li>▪ Full coverage in each set of assessments (but not every assessment).</li> <li>▪ Taken together, strands 1 and 2 should comprise at least 40% of the marks for this assessment objective.</li> </ul>	<ul style="list-style-type: none"> <li>▪ <b>Translate problems</b> may involve the Learner selecting and/or constructing appropriate mathematical processes. It includes identifying important features or variables.</li> <li>▪ <b>Translate situations in context</b> may involve the Learner selecting and/or constructing appropriate mathematical models. It may also entail constructing a model of a specific situation, following the principles of an established modelling process. It includes identifying important features or variables.</li> <li>▪ Within strands 2 and 5, <b>where appropriate</b> means where it is meaningful to do so in the context of the question/task. As such, these abilities need not be targeted on every occasion, but should be covered in every set of assessments.</li> <li>▪ Within strand 2, evaluating the accuracy and limitations of a solution may (but need not) extend to the problem-solving process used in generating it</li> </ul>
	1b – translate problems in non-mathematical contexts into mathematical processes		
<b>2 – interpret solutions in the context of a</b>	This strand is a single element.		

<b>AO3: Solve problems within mathematics and in other contexts</b>			<b>25% (A level)</b>
<b>Learners should be able to:</b> <ul style="list-style-type: none"> <li>▪ translate problems in mathematical and non-mathematical contexts into mathematical processes</li> <li>▪ interpret solutions in the context of a problem, and, where appropriate, evaluate their accuracy and limitations</li> <li>▪ translate situations in context into mathematical models</li> <li>▪ use mathematical models</li> <li>▪ evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them</li> </ul>			<b>20% (AS)</b>
<b>Strands</b>	<b>Elements</b>	<b>Coverage</b>	<b>Interpretation and definitions</b>
<b>problem, and, where appropriate evaluate their accuracy and limitations</b>		<ul style="list-style-type: none"> <li>▪ Taken together, strands 3, 4 and 5 should comprise at least 40% of the marks for this assessment objective.</li> </ul>	<ul style="list-style-type: none"> <li>▪ A problem-solving question/task would typically exhibit<sup>18</sup> one or more of the following attributes –                             <ul style="list-style-type: none"> <li>□ Little or no scaffolding – the Learner receives little guidance beyond a start point and a finish point, and the mathematical processes required for the solution are not explicitly stated.</li> <li>□ Provision for multiple representations (such as the use of a sketch or diagram as well as calculations).</li> <li>□ Information is not given in mathematical form or mathematical language, or results and/or methods need to be interpreted and/or evaluated (for example, in a real-world context).</li> <li>□ A choice of techniques to be used.</li> </ul> </li> </ul>
<b>3 – translate situations in context into mathematical models</b>	This strand is a single element.		

<sup>18</sup> For the avoidance of doubt, individual questions/tasks need not exhibit all of these attributes to be considered problem-solving. Neither does demonstrating one or more of these attributes necessarily indicate that a question/task requires problem-solving.

<b>AO3: Solve problems within mathematics and in other contexts</b>			<b>25% (A level)</b>
<b>Learners should be able to:</b> <ul style="list-style-type: none"> <li>▪ translate problems in mathematical and non-mathematical contexts into mathematical processes</li> <li>▪ interpret solutions in the context of a problem, and, where appropriate, evaluate their accuracy and limitations</li> <li>▪ translate situations in context into mathematical models</li> <li>▪ use mathematical models</li> <li>▪ evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them</li> </ul>			<b>20% (AS)</b>
<b>Strands</b>	<b>Elements</b>	<b>Coverage</b>	<b>Interpretation and definitions</b>
<b>4 – use mathematical models</b>	This strand is a single element.		<ul style="list-style-type: none"> <li>□ The solution requires understanding of the processes involved, rather than just application of techniques.</li> <li>□ Two or more mathematical processes are required, or the solution requires drawing together different parts of mathematics.</li> <li>■ Each set of assessments should include questions/tasks where Learners are assessed on their ability to solve complete problems presented in an unstructured manner and which require the use of multiple parts of the problem-solving cycle (as defined in overarching theme 2 on pages 5–6 of the Content Document).</li> <li>■ Within each set of assessments, there should be opportunities for both –                             <ul style="list-style-type: none"> <li>□ extended questions/tasks that address strands 1 and 2 in combination, and</li> <li>□ extended questions/tasks that address strands 3, 4 and 5 in combination.</li> </ul> </li> </ul>
<b>5 – evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them</b>	5a – evaluate the outcomes of modelling in the context of a problem 5b – recognise the limitations of models, and, where appropriate, explain how to refine them		

## Proposed guidance for AS and A level further mathematics

4.6 We are proposing to introduce the following guidance which will apply to all new AS and A level qualifications in further mathematics:

- guidance on interpretation of the subject content;
- guidance on comparability of optional routes; and
- guidance on assessment objectives.<sup>19</sup>

4.7 Because mathematics and further mathematics have the same assessment objectives, our proposed guidance on assessment objectives for further mathematics is almost identical to the equivalent proposed guidance for mathematics. The only differences are:

- the references to the weightings of AO2 and AO3 (which reflect the different approach we are taking to these weightings in further mathematics; and
- within AO3, the suggested minimum weightings for strands 1 and 2, and for strands 3, 4 and 5 are lower – this reflects the fact that the optimum weighting for these strands will depend on each exam board’s choice of non-core content. We are also asking exam boards to explain the weighting they have assigned to these strands in their assessment strategies.

### **Guidance in relation to subject content for GCE Qualifications in Further Mathematics**

The subject content for GCE Qualifications in Further Mathematics is set out in the Department for Education’s *Further mathematics AS and A level content*, document reference DFE-00707-2014 (the ‘Content Document’).

Condition GCE(Further Mathematics)1.1(c) requires awarding organisations to interpret the Content Document in line with any requirements, and have regard to any guidance, published by Ofqual.

We set out our guidance for the purposes of Condition GCE(Further Mathematics)1.1(c) below.

#### **Overarching themes and use of technology**

Paragraphs 10 and 11 of the Content Document state that –

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<sup>19</sup> Aside from the weightings of the assessment objectives, the proposed guidance on assessment objectives are common across mathematics and further mathematics.

10. A level specifications in further mathematics must require students to demonstrate the following overarching knowledge and skills. These must be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content set out below. The knowledge and skills are similar to those specified for A level mathematics but they will be examined against further mathematics content and contexts.

[list of overarching themes]

11. The use of technology, in particular mathematical graphing tools and spreadsheets, must permeate the study of AS and A level further mathematics.

These statements should be interpreted primarily as indicating the desired approach to teaching GCE Qualifications in Further Mathematics.

However, these statements also have implications for assessments. Consequently, in respect of each GCE Qualification in Further Mathematics which it makes available, or proposes to make available, we expect an awarding organisation to explain and justify in its assessment strategy for that qualification how these requirements have been reflected in the qualification's design.

### **Guidance on comparability of optional routes through GCE Qualifications in Further Mathematics**

The subject content for GCE Qualifications in Further Mathematics is set out in the Department for Education's *Further mathematics AS and A level content*, document reference DFE-00707-2014 (the 'Content Document').

The Content Document allows an awarding organisation to develop a GCE Qualification in Further Mathematics which has a number of optional routes through it. Ofqual is particularly mindful of the risks to the comparability of standards that may arise if this results in a large number of optional routes.

General Condition G1.2 states that an awarding organisation must take all reasonable steps to ensure that the Level of Demand of an assessment for a qualification it makes available is consistent across all options as to tasks and alternative assessments which may be taken by the Learner for the purpose of the qualification.

Ofqual's *GCE Subject Level Conditions and Requirements for Further Mathematics* also require an awarding organisation to –

explain and justify in its assessment strategy for the qualification –  
 [...] how it will secure Comparability, including over time and between any optional routes through the qualification.

Ofqual's *GCE Qualification Level Guidance*<sup>20</sup> states, in relation to Condition GCE2 (Assessment Strategies), that each awarding organisation's assessment strategy should explain how –

optional routes through the qualification [...] will be comparable in terms of the Level of Demand and the amount of subject content required to be taught and on which Learners will be assessed (D1.1, D1.2(c), G1.2).

We expect an awarding organisation to be able to demonstrate in its assessment strategy for a GCE Qualification in Further Mathematics that it has taken all reasonable steps to ensure that it will have maximum control over comparability of standards between different routes at all points within the cycle of setting and marking assessments and awarding the qualification. In particular, this control should be such that smaller entry routes are neither advantaged nor disadvantaged.

### Guidance on assessment objectives for GCE Qualifications in Further Mathematics

Condition GCE(Further Mathematics)1.2 allows us to specify requirements and guidance relating to assessment objectives for GCE Qualifications in Further Mathematics.

We published our requirements in relation to assessment objectives in *GCE Subject Level Conditions and Requirements for Further Mathematics*, and reproduce them in the table below.

	Objective	Weighting	
		A level	AS
<b>AO1</b>	<b>Use and apply standard techniques</b> Learners should be able to: <ul style="list-style-type: none"> <li>■ select and correctly carry out routine procedures; and</li> </ul>	50% (±2%)	60% (±2%)

<sup>20</sup> [www.gov.uk/government/publications/gce-qualification-level-guidance](http://www.gov.uk/government/publications/gce-qualification-level-guidance)



	<ul style="list-style-type: none"> <li>■ accurately recall facts, terminology and definitions.</li> </ul>		
<b>AO2</b>	<p><b>Reason, interpret and communicate mathematically</b></p> <p>Learners should be able to:</p> <ul style="list-style-type: none"> <li>■ construct rigorous mathematical arguments (including proofs);</li> <li>■ make deductions and inferences;</li> <li>■ assess the validity of mathematical arguments;</li> <li>■ explain their reasoning; and</li> <li>■ use mathematical language correctly.</li> </ul> <p><i>Where questions/tasks targeting this assessment objective will also credit Learners for the ability to ‘use and apply standard techniques’ (AO1) and/or to ‘solve problems within mathematics and other contexts’ (AO3) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).</i></p>	At least 15%	At least 10%
<b>AO3</b>	<p><b>Solve problems within mathematics and in other contexts</b></p> <p>Learners should be able to:</p> <ul style="list-style-type: none"> <li>■ translate problems in mathematical and non-mathematical contexts into mathematical processes;</li> <li>■ interpret solutions in the context of a problem, and, where appropriate, evaluate their accuracy and limitations;</li> <li>■ translate situations in context into mathematical models;</li> <li>■ use mathematical models; and</li> <li>■ evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them.</li> </ul> <p><i>Where questions/tasks targeting this assessment objective will also credit Learners for the ability to ‘use and apply standard techniques’ (AO1) and/or to ‘reason, interpret and communicate mathematically’ (AO2) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).</i></p>	At least 15%	At least 10%

We set out below our guidance for the purposes of Condition GCE(Further Mathematics)1.2. This guidance explains how we expect awarding organisations to interpret these assessment objectives in terms of:

- the different ‘strands’ within each of the assessment objectives;
- the discrete ‘elements’ within each assessment objective and its strands which questions and tasks could target and/or seek to credit – our expectation is that each and every question/task should target or seek to credit at least one of these elements, and may target or seek to credit multiple elements across one or more assessment objectives;
- the coverage expectations, such as in relation to the different elements within each assessment objective and how those elements should be sampled over time; and
- the key areas of emphasis in each assessment objective and the particular meaning for the subject of any key terms and phrases used; defined terms are shown in bold text, followed by their definitions.

In line with the obligations set out in Condition GCE(Further Mathematics)1.2, we expect awarding organisations to be able to demonstrate how they have had regard to this guidance. For example, an awarding organisation could map how it has regard to the guidance as it:

- develops its sample assessment materials;
- delivers the qualification;
- develops and applies its approach to sampling the elements into which the assessment objectives are divided; and
- monitors the qualification to make sure it addresses all elements appropriately.

AO1: Use and apply standard techniques			50% (A level)
Learners should be able to:			60% (AS)
<ul style="list-style-type: none"> <li>▪ select and correctly carry out routine procedures</li> <li>▪ accurately recall facts, terminology and definitions</li> </ul>			
Strands	Elements	Coverage	Interpretation and definitions
1 – select and correctly carry out routine procedures	1a – select routine procedures	<ul style="list-style-type: none"> <li>▪ Full coverage in each set of assessments (but not every assessment).</li> <li>▪ No more than 10% of the marks for this assessment objective should be allocated solely to strand 2.</li> </ul>	<ul style="list-style-type: none"> <li>▪ <b>Routine procedures</b> includes multi-step as well as single-step processes. They should be familiar to the Learner (including, but not limited to, those stated in the specification) and there should be no significant background context given in the question/task that would have an impact on the Level of Demand.</li> <li>▪ <b>Select</b> involves the recognition of a single- or multi-step process necessary to carry out a routine procedure (for example, solving a quadratic equation or integrating a function) in cases where the question/task does not make the required process clear. It should not be confused with the more complex decision-making required in AO3.</li> <li>▪ Element 1a should normally be assessed in combination with element 1b.</li> <li>▪ Within strand 2, individual questions/tasks may target each of facts, terminology and definitions in isolation, or in any combination.</li> </ul>
	1b – correctly carry out routine procedures		
2 – accurately recall facts, terminology and definitions	This strand is a single element.		

AO2: Reason, interpret and communicate mathematically			At least 15% (A level)
Learners should be able to: <ul style="list-style-type: none"> <li>▪ construct rigorous mathematical arguments (including proofs)</li> <li>▪ make deductions and inferences</li> <li>▪ assess the validity of mathematical arguments</li> <li>▪ explain their reasoning</li> <li>▪ use mathematical language correctly</li> </ul>			At least 10% (AS)
Strands	Elements	Coverage	Interpretation and definitions
<b>1 – construct rigorous mathematical arguments (including proofs)</b>	This strand is a single element.	<ul style="list-style-type: none"> <li>▪ Full coverage in each set of assessments (but not every assessment).</li> <li>▪ Taken together, strands 1 and 2 should comprise at least 50% of the marks for this assessment objective.</li> </ul>	<ul style="list-style-type: none"> <li>▪ A <b>mathematical argument</b> leads from premises to a conclusion, through rigorous and clear reasoning. It:                             <ul style="list-style-type: none"> <li>□ may include one or both of calculations or algebraic manipulation, but is more than these alone,</li> <li>□ typically involves several steps, which are logical in nature and sequence, given the context,</li> <li>□ clearly identifies the principal results used where appropriate, and</li> <li>□ contains sufficient detail to allow the line of reasoning to be followed.</li> </ul> </li> <li>▪ <b>Deduction</b> means a process of reasoning from absolutes to results that must be correct.</li> <li>▪ <b>Inference</b> means a process of reasoning from relative or partial evidence to results that are likely to be correct.</li> <li>▪ Strand 3 could apply to arguments provided to a Learner, or to arguments generated by them. It may include identifying:                             <ul style="list-style-type: none"> <li>□ errors and/or omissions, and</li> </ul> </li> </ul>
<b>2 – make deductions and inferences</b>	2a – make deductions 2b – make inferences		
<b>3 – assess the validity of mathematical arguments</b>	This strand is a single element.	<ul style="list-style-type: none"> <li>▪ No more than 10% of the marks for this assessment objective should</li> </ul>	
<b>4 – explain their reasoning</b>	This strand is a single element.		

<b>AO2: Reason, interpret and communicate mathematically</b>			At least 15% (A level)
Learners should be able to:			At least 10% (AS)
<ul style="list-style-type: none"> <li>▪ construct rigorous mathematical arguments (including proofs)</li> <li>▪ make deductions and inferences</li> <li>▪ assess the validity of mathematical arguments</li> <li>▪ explain their reasoning</li> <li>▪ use mathematical language correctly</li> </ul>			
Strands	Elements	Coverage	Interpretation and definitions
<b>5 – use mathematical language correctly</b>	This strand is a single element.	be allocated to strand 3. <ul style="list-style-type: none"> <li>▪ No more than 10% of the marks for this assessment objective should be allocated solely to strand 5.</li> </ul>	<ul style="list-style-type: none"> <li>□ the values for, and/or conditions under, which an argument remains correct.</li> <li>▪ In the context of strand 4, <b>explain</b> may include providing justification for a particular approach. Justification and/or explanation of key steps in the working should be required even where problems are otherwise fairly routine in nature.</li> <li>▪ The correct use of mathematical notation is included within strand 5.</li> <li>▪ Strands 4 and 5 should normally be assessed in combination with strands 1, 2 and/or 3. Learners should be given multiple opportunities to demonstrate (and gain credit for) the skills required by these strands.</li> </ul>

<b>AO3: Solve problems within mathematics and in other contexts</b>			<i>At least 15% (A level)</i>
<b>Learners should be able to:</b> <ul style="list-style-type: none"> <li>▪ translate problems in mathematical and non-mathematical contexts into mathematical processes</li> <li>▪ interpret solutions in the context of a problem, and, where appropriate, evaluate their accuracy and limitations</li> <li>▪ translate situations in context into mathematical models</li> <li>▪ use mathematical models</li> <li>▪ evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them</li> </ul>			<i>At least 10% (AS)</i>
Strands	Elements	Coverage	Interpretation and definitions
<b>1 – translate problems in mathematical and non-mathematical contexts into mathematical processes</b>	1a – translate problems in mathematical contexts into mathematical processes	<ul style="list-style-type: none"> <li>▪ Full coverage in each set of assessments (but not every assessment).</li> <li>▪ Taken together, strands 1 and 2 should comprise at least 20% of the marks for this assessment objective.</li> </ul>	<ul style="list-style-type: none"> <li>▪ <b>Translate problems</b> may involve the Learner selecting and/or constructing appropriate mathematical processes. It includes identifying important features or variables.</li> <li>▪ <b>Translate situations in context</b> may involve the Learner selecting and/or constructing appropriate mathematical models. It may also entail constructing a model of a specific situation, following the principles of an established modelling process. It includes identifying important features or variables.</li> <li>▪ Within strands 2 and 5, <b>where appropriate</b> means where it is meaningful to do so in the context of the question/task. As such, these abilities need not be targeted on every occasion, but should be covered in every set of assessments.</li> <li>▪ Within strand 2, evaluating the accuracy and limitations of a solution may (but need not) extend to the problem-solving process used in generating it</li> </ul>
	1b – translate problems in non-mathematical contexts into mathematical processes		
<b>2 – interpret solutions in the context of a</b>	This strand is a single element.		

<b>AO3: Solve problems within mathematics and in other contexts</b>			<i>At least 15% (A level)</i>
<b>Learners should be able to:</b> <ul style="list-style-type: none"> <li>▪ translate problems in mathematical and non-mathematical contexts into mathematical processes</li> <li>▪ interpret solutions in the context of a problem, and, where appropriate, evaluate their accuracy and limitations</li> <li>▪ translate situations in context into mathematical models</li> <li>▪ use mathematical models</li> <li>▪ evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them</li> </ul>			<i>At least 10% (AS)</i>
Strands	Elements	Coverage	Interpretation and definitions
<b>problem, and, where appropriate, evaluate their accuracy and limitations</b>		<ul style="list-style-type: none"> <li>▪ Taken together, strands 3, 4 and 5 should comprise at least 20% of the marks for this assessment objective.</li> </ul>	<ul style="list-style-type: none"> <li>▪ A problem-solving question/task would typically exhibit<sup>21</sup> one or more of the following attributes –                             <ul style="list-style-type: none"> <li>□ Little or no scaffolding – the Learner receives little guidance beyond a start point and a finish point, and the mathematical processes required for the solution are not explicitly stated.</li> <li>□ Provision for multiple representations (such as the use of a sketch or diagram as well as calculations).</li> <li>□ Information is not given in mathematical form or mathematical language, or results and/or methods need to be interpreted and/or evaluated (for example, in a real-world context).</li> <li>□ A choice of techniques to be used.</li> </ul> </li> </ul>
<b>3 – translate situations in context into mathematical models</b>	This strand is a single element.		

<sup>21</sup> For the avoidance of doubt, individual questions/tasks need not exhibit all of these attributes to be considered problem-solving. Neither does demonstrating one of more of these attributes necessarily indicate that a question/task requires problem-solving.

<b>AO3: Solve problems within mathematics and in other contexts</b>			<i>At least 15% (A level)</i>
<b>Learners should be able to:</b> <ul style="list-style-type: none"> <li>▪ translate problems in mathematical and non-mathematical contexts into mathematical processes</li> <li>▪ interpret solutions in the context of a problem, and, where appropriate, evaluate their accuracy and limitations</li> <li>▪ translate situations in context into mathematical models</li> <li>▪ use mathematical models</li> <li>▪ evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them</li> </ul>			<i>At least 10% (AS)</i>
Strands	Elements	Coverage	Interpretation and definitions
		<ul style="list-style-type: none"> <li>▪ Awarding organisations should explain in their assessment strategies why the weightings assigned to strands 1 and 2, and to strands 3, 4, and 5 are appropriate, including for any optional routes through the qualification</li> </ul>	<ul style="list-style-type: none"> <li>□ The solution requires understanding of the processes involved, rather than just application of techniques.</li> <li>□ Two or more mathematical processes are required, or the solution requires drawing together different parts of mathematics.</li> </ul>
<b>4 – use mathematical models</b>	This strand is a single element		
<b>5 – evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them</b>	5a – evaluate the outcomes of modelling in the context of a problem		<ul style="list-style-type: none"> <li>▪ Each set of assessments should include questions/tasks where Learners are assessed on their ability to solve complete problems presented in an unstructured manner and which require the use of multiple parts of the problem-solving cycle (as defined in overarching theme 2 on pages 5–6 of the Content Document).</li> <li>▪ Within each set of assessments, there should be opportunities for both –                             <ul style="list-style-type: none"> <li>□ extended questions/tasks that address strands 1 and 2 in combination, and</li> <li>□ extended questions/tasks that address strands 3, 4 and 5 in combination.</li> </ul> </li> </ul>
	5b – recognise the limitations of models, and, where appropriate, explain how to refine them		



## **Questions on proposed Conditions, requirements and guidance**

**Question 13: Do you have any comments on our proposed Conditions and requirements for AS and A level mathematics?**

**Question 14: Do you have any comments on our proposed Conditions and requirements for AS and A level further mathematics?**

**Question 15: Do you have any comments on our proposed guidance for AS and A level mathematics?**

**Question 16: Do you have any comments on our proposed guidance for AS and A level further mathematics?**

## DfE’s proposed appendices to the subject content

- 5.1 DfE is proposing to add two new appendices to the subject content for new AS and A levels in mathematics and further mathematics:
- one covering mathematical notation (this will be the same for mathematics and further mathematics); and
  - one covering mathematical formulae that students must not be given in exams (there are separate appendices for mathematics and further mathematics).
- 5.2 The draft text for these appendices has been set out in Appendix C to this consultation.
- 5.3 We are consulting on these changes on behalf of DfE so that stakeholders can see all the proposals for AS and A level mathematics and further mathematics in one place, and need only respond to a single consultation.
- 5.4 These proposals are based on the equivalent rules for current AS and A level qualifications in mathematics, further mathematics and pure mathematics (set out in the appendices to our document *GCE AS and A Level Subject Criteria for Mathematics*<sup>22</sup>), modified to reflect the different subject content requirements for the new AS and A level qualifications.
- 5.5 DfE will consider responses to this consultation on the proposed new appendices before taking final decisions on whether (and, if so, how) to amend the subject content. If DfE does decide to include the new appendices they will be covered by our condition which requires exam boards to comply with the subject content.

### Questions on proposed changes to subject content

**Question 17: Do you have any comments on DfE’s proposed new appendices to the subject content for mathematics and further mathematics?**

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<sup>22</sup> [www.gov.uk/government/publications/gce-as-and-a-level-subject-criteria-for-mathematics](http://www.gov.uk/government/publications/gce-as-and-a-level-subject-criteria-for-mathematics)

## Equality impact analysis

### Ofqual's role, objectives and duties

6.1 We are subject to the public sector equality duty. We have set out in Appendix D how this duty interacts with our statutory objectives and other duties.

### Equality impact analysis relating to proposed changes to AS and A level mathematics and further mathematics

6.2 We have considered the potential impact on students who share protected characteristics<sup>23</sup> of the application of the principles and features that will apply to all new AS and A level qualifications. Our equality impact analysis for our earlier consultation on AS and A level reform<sup>24</sup> is therefore of interest and we encourage you to read it.

6.3 Issues concerning the proposed subject content have been considered by DfE, who have published their own Equalities Impact Analysis on their subject content proposals.<sup>25</sup>

6.4 We have also previously considered the potential impact on students who share protected characteristics of our decision that mathematics and further mathematics should be assessed wholly by exam.<sup>26</sup>

6.5 We do not repeat here all of the evidence we have considered, as this can be found in our earlier reports. We focus instead on the specific issues that arise from the new proposals in this consultation, and from the way in which we are implementing our previous policy decisions.

6.6 We have not identified any additional negative impacts on students who share protected characteristics which would result from the proposals in this consultation (beyond those that we and DfE have already identified in our earlier reports).

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<sup>23</sup> For the purposes of the public sector equality duty, the protected characteristics are disability, racial group, age, religion or belief, pregnancy or maternity, sex, sexual orientation, gender reassignment.

<sup>24</sup> <http://webarchive.nationalarchives.gov.uk/20141031163546/http://www.ofqual.gov.uk/files/2012-06-18-equality-analysis-of-the-a-level-reform-consultation.pdf>

<sup>25</sup> [www.gov.uk/government/publications/a-level-subject-content-equality-impact-assessment](http://www.gov.uk/government/publications/a-level-subject-content-equality-impact-assessment)

<sup>26</sup> [www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/389439/ea-developing-new-gcse-a-level-and-as-qualifications-for-first-teaching-in-2016.pdf](http://www.gov.uk/government/uploads/system/uploads/attachment_data/file/389439/ea-developing-new-gcse-a-level-and-as-qualifications-for-first-teaching-in-2016.pdf)

- 6.7 During this consultation, we will continue to seek and consider evidence and feedback to our proposals that might help us identify any potential subject-specific impacts on students who share a protected characteristic.
- 6.8 Exam boards are required to consider the accessibility of their qualifications at the design stage and to remove any unjustifiable barriers.

**Question 18: We have not identified any ways in which the proposals for AS and A level mathematics and further mathematics would impact (positively or negatively) on persons who share a protected characteristic.<sup>27</sup> Are there any potential impacts we have not identified?**

**Question 19: Are there any additional steps we could take to mitigate any negative impact resulting from these proposals on persons who share a protected characteristic?**

**Question 20: Have you any other comments on the impacts of the proposals on students who share a protected characteristic?**

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<sup>27</sup> 'Protected characteristic' is defined in the Equality Act 2010. Here, it means disability, racial group, age, religion or belief, pregnancy or maternity, sex, sexual orientation and gender reassignment.

## Appendix A: Regulatory tools

### Comparability and innovation

Exam boards operate in a market. They can design and deliver their qualifications in different ways, within the parameters we set. This provides some choice to schools or colleges, which is one of the benefits of a qualifications market. Exam boards must, however, make sure that the levels of attainment indicated by their qualifications are comparable to those of other exam boards' versions of the qualifications. The exam boards cooperate in a range of ways to make sure that the standards of their respective qualifications are comparable. To make sure standards are maintained and comparability is secured, we review AS and A level qualifications before they can be made available, by applying an accreditation requirement to the qualifications, and we oversee the awarding of AS and A level qualifications.

We do not wish to close down opportunities for exam boards to design and deliver their qualifications in different ways. Indeed, we have a statutory duty to have regard to the desirability of facilitating innovation in connection with the provision of regulated qualifications and a statutory objective with regard to the efficiency with which the qualifications market works. If we adopt a regulatory approach in which all aspects of a qualification are very tightly defined, we could effectively remove scope for exam boards to distinguish their qualifications from others and stop choice for schools or colleges. On the other hand, if exam boards have too much scope to vary their approach their qualifications might not be comparable.

In striking a balance, we use a range of tools to regulate qualifications and the exam boards that provide them. The main regulatory tools we use for the qualifications in this consultation are explained below.

### Conditions of Recognition

Exam boards must comply at all times with our Conditions of Recognition. These are the main regulatory rules that we use. We can take regulatory action against an exam board that breaches or is likely to breach a Condition.

There are three sets of Conditions that will apply to new AS and A level qualifications:

- (i) the published *General Conditions of Recognition*<sup>28</sup> that apply to all regulated qualifications;

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<sup>28</sup> [www.gov.uk/government/publications/general-conditions-of-recognition](http://www.gov.uk/government/publications/general-conditions-of-recognition)

- (ii) *GCE Qualification Level Conditions and Requirements*<sup>29</sup> that apply to all new AS and A level qualifications;
- (iii) GCE Subject Level Conditions that apply to new AS and A level qualifications in a specific subject. We are consulting now on two sets of draft GCE Subject Level Conditions – one covering AS and A level mathematics, and one covering AS and A level further mathematics.

## **Regulatory documents**

In some Conditions we refer to published regulatory requirements. We publish these in regulatory documents. The Conditions require exam boards to comply with such documents.

We are proposing two separate sets of regulatory documents – one set for mathematics, and one for further mathematics. These documents cover our detailed requirements in relation to:

- interpretation of subject content in further mathematics; and
- assessments in mathematics.

The requirements will have effect as if they were part of a Condition. The requirements will be set out in a stand-alone section of the Conditions document, simply because they are technical and detailed so they sit better as separate from, rather than within, the Condition itself.

## **Statutory guidance**

We publish guidance to help exam boards identify the types of behaviour or practices they could use to meet a Condition. Exam boards must have regard to such guidance, but they do not have to follow this guidance in the same way that they must comply with the Conditions; they are free to meet the outcomes of the Conditions in their own ways. An exam board that decides to take a different approach to that set out in our guidance must still be able to show that it is meeting the Condition or Conditions to which the guidance relates.

We are consulting now on two separate sets of draft guidance – one for mathematics, and one for further mathematics.

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<sup>29</sup> [www.gov.uk/government/publications/gce-qualification-level-conditions-and-requirements](http://www.gov.uk/government/publications/gce-qualification-level-conditions-and-requirements)

## Appendix B: Previously proposed assessment objectives

We set out below the assessment objectives that we previously proposed in our *Developing new GCSE, A level and AS qualifications for first teaching in 2016* consultation. The same assessment objectives and weightings were proposed for both mathematics and further mathematics:

Assessment Objective	Weighting	
	A level	AS
<p><b>AO1 – Use and apply standard techniques</b> Students should be able to:</p> <ul style="list-style-type: none"> <li>■ accurately recall facts, terminology, definitions and proofs;</li> <li>■ use and interpret notation correctly; and</li> <li>■ accurately carry out routine procedures or set tasks requiring multi-step solutions.</li> </ul>	30–40%	35–45%
<p><b>AO2 – Reason, interpret and communicate mathematically</b> Students should be able to:</p> <ul style="list-style-type: none"> <li>■ independently construct a rigorous, non-standard proof or mathematical argument;</li> <li>■ construct extended chains of reasoning to achieve a given result; and</li> <li>■ assess, critique and improve the validity of a mathematical argument, making deductions and inferences, finding and correcting errors in reasoning and evaluating evidence.</li> </ul> <p><i>Where problems require candidates to ‘use and apply standard techniques’ or to ‘solve problems’ independently a proportion of those marks should be attributed to the corresponding assessment objective.</i></p>	30–40%	30–40%
<p><b>AO3 – Solve problems within mathematics and in other contexts</b> Learners should be able to:</p> <ul style="list-style-type: none"> <li>■ translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes, identifying important features or variables and using appropriate techniques;</li> <li>■ make and use connections between different parts of mathematics;</li> <li>■ evaluate methods used and solutions obtained, recognising limitations and sources of error;</li> </ul>	30–40%	25–35%

<ul style="list-style-type: none"><li>■ construct, select and refine mathematical models; and</li><li>■ interpret the outcomes of a modelling process in real world terms and recognise the limitations of a model.</li></ul> <p><i>Where problems require candidates to ‘use and apply standard techniques’ or to ‘reason, interpret and communicate mathematically’ a proportion of those marks should be attributed to the corresponding assessment objective.</i></p>		
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## **Appendix C: DfE’s proposed appendices to the subject content**

## Mathematical notation for AS/A levels in Mathematics and Further Mathematics

1	Set Notation (Mathematics)	
1.1	$\in$	is an element of
1.2	$\notin$	is not an element of
1.3	$\subseteq$	is a subset of
1.4	$\subset$	is a proper subset of
1.5	$\{x_1, x_2, \dots\}$	the set with elements $x_1, x_2, \dots$
1.6	$\{x: \dots\}$	the set of all $x$ such that ...
1.7	$n(A)$	the number of elements in set $A$
1.8	$\emptyset$	the empty set
1.9	$\mathcal{E}$	the universal set
1.10	$A'$	the complement of the set $A$
1.11	$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \dots\}$
1.12	$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
1.13	$\mathbb{R}$	the set of real numbers
1.14	$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \dots\}$
1.15	$\mathbb{Z}_0^+$	the set of positive integers and 0, $\{0, 1, 2, 3, \dots\}$
1.16	$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
1.17	$\cup$	union
1.18	$\cap$	intersection
1.19	$(x, y)$	the ordered pair $x, y$
1.20	$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$
1.21	$[a, b)$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
1.22	$(a, b]$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
1.23	$(a, b)$	the open interval $\{x \in \mathbb{R} : a < x < b\}$

<b>1</b>	<b>Set Notation (Further Mathematics)</b>	
1.24	$\mathbb{C}$	the set of complex numbers
<b>2</b>	<b>Miscellaneous Symbols (Mathematics)</b>	
2.1	=	is equal to
2.2	$\neq$	is not equal to
2.3	$\equiv$	is identical to or is congruent to
2.4	$\approx$	is approximately equal to
2.5	$\infty$	infinity
2.6	$\propto$	is proportional to
2.7	$\therefore$	therefore
2.8	$\because$	because
2.9	<	is less than
2.10	$\leq, \leq$	is less than or equal to, is not greater than
2.11	>	is greater than
2.12	$\geq, \geq$	is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	$p$ implies $q$ (if $p$ then $q$ )
2.14	$p \Leftarrow q$	$p$ is implied by $q$ (if $q$ then $p$ )
2.15	$p \Leftrightarrow q$	$p$ implies and is implied by $q$ ( $p$ is equivalent to $q$ )
2.16	$a$	first term for an arithmetic or geometric sequence
2.17	$l$	last term for an arithmetic sequence
2.18	$d$	common difference for an arithmetic sequence
2.19	$r$	common ratio for a geometric sequence
2.20	$S_n$	sum to $n$ terms of a sequence
2.21	$S_\infty$	sum to infinity of a sequence
<b>3</b>	<b>Operations (Mathematics)</b>	
3.1	$a + b$	$a$ plus $b$
3.2	$a - b$	$a$ minus $b$
3.3	$a \times b, ab, a.b$	$a$ multiplied by $b$
3.4	$a \div b, \frac{a}{b}$	$a$ divided by $b$

3.5	$\sum_{i=1}^n a_i$	$a_1 + a_2 + \dots + a_n$
3.6	$\prod_{i=1}^n a_i$	$a_1 \times a_2 \times \dots \times a_n$
3.7	$\sqrt{a}$	the positive square root of $a$
3.8	$ a $	the modulus of $a$
3.9	$n!$	$n$ factorial: $n! = n \times (n-1) \dots \times 2 \times 1$
3.10	$\binom{n}{r}, {}^n C_r, {}_n C_r$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}_0^+, r \leq n$ or $\frac{n(n-1)\dots(n-r+1)}{r!}$ for $n \in \mathbb{Q}, r \in \mathbb{Z}_0^+$
<b>4</b>	<b>Functions (Mathematics)</b>	
4.1	$f(x)$	the value of the function $f$ at $x$
4.2	$f : x \mapsto y$	the function $f$ maps the element $x$ to the element $y$
4.3	$f^{-1}$	the inverse function of the function $f$
4.4	$gf$	the composite function of $f$ and $g$ which is defined by $gf(x) = g(f(x))$
4.5	$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as $x$ tends to $a$
4.6	$\Delta x, \delta x$	an increment of $x$
4.7	$\frac{dy}{dx}$	the derivative of $y$ with respect to $x$
4.8	$\frac{d^n y}{dx^n}$	the $n$ th derivative of $y$ with respect to $x$
4.9	$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ..., $n$ th derivatives of $f(x)$ with respect to $x$
4.10	$\dot{x}, \ddot{x}, \dots$	the first, second, ... derivatives of $x$ with respect to $t$
4.11	$\int y \, dx$	the indefinite integral of $y$ with respect to $x$
4.12	$\int_a^b y \, dx$	the definite integral of $y$ with respect to $x$ between the limits $x = a$ and $x = b$
<b>5</b>	<b>Exponential and Logarithmic Functions (Mathematics)</b>	
5.1	$e$	base of natural logarithms
5.2	$e^x, \exp x$	exponential function of $x$

5.3	$\log_a x$	logarithm to the base $a$ of $x$
5.4	$\ln x, \log_e x$	natural logarithm of $x$
<b>6</b>	<b>Trigonometric and Hyperbolic Functions (Mathematics)</b>	
6.1	$\left. \begin{array}{l} \sin, \cos, \tan, \\ \operatorname{cosec}, \sec, \cot \end{array} \right\}$	the trigonometric functions
6.2	$\left. \begin{array}{l} \sin^{-1}, \cos^{-1}, \tan^{-1} \\ \operatorname{arcsin}, \operatorname{arccos}, \operatorname{arctan} \end{array} \right\}$	the inverse trigonometric functions
6.3	$^\circ$	degrees
6.4	rad	radians
<b>6</b>	<b>Trigonometric and Hyperbolic Functions (Further Mathematics)</b>	
6.5	$\left. \begin{array}{l} \operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1} \\ \operatorname{arcosec}, \operatorname{arcsec}, \operatorname{arccot} \end{array} \right\}$	the inverse trigonometric functions
6.6	$\left. \begin{array}{l} \sinh, \cosh, \tanh, \\ \operatorname{cosech}, \operatorname{sech}, \operatorname{coth} \end{array} \right\}$	the hyperbolic functions
6.7	$\left. \begin{array}{l} \sinh^{-1}, \cosh^{-1}, \tanh^{-1} \\ \operatorname{cosech}^{-1}, \operatorname{sech}^{-1}, \operatorname{coth}^{-1} \\ \operatorname{arsinh}, \operatorname{arcosh}, \operatorname{artanh}, \\ \operatorname{arcosech}, \operatorname{arcsech}, \operatorname{arcoth} \end{array} \right\}$	the inverse hyperbolic functions
<b>7</b>	<b>Complex Numbers (Further Mathematics)</b>	
7.1	$i, j$	square root of $-1$
7.2	$x + iy$	complex number with real part $x$ and imaginary part $y$
7.3	$r(\cos \theta + i \sin \theta)$	modulus argument form of a complex number with modulus $r$ and argument $\theta$
7.4	$z$	a complex number, $z = x + iy = r(\cos \theta + i \sin \theta)$
7.5	$\operatorname{Re}(z)$	the real part of $z$ , $\operatorname{Re}(z) = x$
7.6	$\operatorname{Im}(z)$	the imaginary part of $z$ , $\operatorname{Im}(z) = y$
7.7	$ z $	the modulus of $z$ , $ z  = \sqrt{x^2 + y^2}$
7.8	$\arg(z)$	the argument of $z$ , $\arg(z) = \theta, -\pi < \theta \leq \pi$
7.9	$z^*$	the complex conjugate of $z$ , $x - iy$

<b>8</b>	<b>Matrices (Further Mathematics)</b>	
8.1	<b>M</b>	a matrix <b>M</b>
8.2	<b>0</b>	zero matrix
8.3	<b>I</b>	identity matrix
8.4	<b>M</b> <sup>-1</sup>	the inverse of the matrix <b>M</b>
8.5	<b>M</b> <sup>T</sup>	the transpose of the matrix <b>M</b>
8.6	$\Delta$ , $\det \mathbf{M}$ or $ \mathbf{M} $	the determinant of the square matrix <b>M</b>
8.7	<b>Mr</b>	Image of column vector <b>r</b> under the transformation associated with the matrix <b>M</b>
<b>9</b>	<b>Vectors (Mathematics)</b>	
9.1	<b>a</b> , $\underline{a}$ , $\hat{a}$	the vector <b>a</b> , $\underline{a}$ , $\hat{a}$
9.2	$\overline{AB}$	the vector represented in magnitude and direction by the directed line segment <b>AB</b>
9.3	$\hat{a}$	a unit vector in the direction of <b>a</b>
9.4	<b>i</b> , <b>j</b> , <b>k</b>	unit vectors in the directions of the cartesian coordinate axes
9.5	$ \mathbf{a} $ , $a$	the magnitude of <b>a</b>
9.6	$ \overline{AB} $ , $AB$	the magnitude of $\overline{AB}$
9.7	$\begin{pmatrix} a \\ b \end{pmatrix}$ , $a\mathbf{i} + b\mathbf{j}$	column vector and corresponding unit vector notation
9.8	<b>r</b>	position vector
9.9	<b>s</b>	displacement vector
9.10	<b>v</b>	velocity vector
9.11	<b>a</b>	acceleration vector
<b>9</b>	<b>Vectors (Further Mathematics)</b>	
9.12	<b>a.b</b>	the scalar product of <b>a</b> and <b>b</b>
<b>10</b>	<b>Differential Equations (Further Mathematics)</b>	
10.1	$\omega$	angular frequency
<b>11</b>	<b>Probability and Statistics (Mathematics)</b>	
11.1	$A, B, C$ , etc.	events
11.2	$A \cup B$	union of the events $A$ and $B$
11.3	$A \cap B$	intersection of the events $A$ and $B$

11.4	$P(A)$	probability of the event $A$
11.5	$A'$	complement of the event $A$
11.6	$P(A B)$	probability of the event $A$ conditional on the event $B$
11.7	$X, Y, R, \text{ etc.}$	random variables
11.8	$x, y, r, \text{ etc.}$	values of the random variables $X, Y, R$ etc.
11.9	$x_1, x_2, \dots$	observations
11.10	$f_1, f_2, \dots$	frequencies with which the observations $x_1, x_2, \dots$ occur
11.11	$p(x), P(X = x)$	probability function of the discrete random variable $X$
11.12	$p_1, p_2, \dots$	probabilities of the values $x_1, x_2, \dots$ of the discrete random variable $X$
11.13	$E(X)$	expectation of the random variable $X$
11.14	$\text{Var}(X)$	variance of the random variable $X$
11.15	$\sim$	has the distribution
11.16	$B(n, p)$	binomial distribution with parameters $n$ and $p$ , where $n$ is the number of trials and $p$ is the probability of success in a trial
11.17	$q$	$q = 1 - p$ for binomial distribution
11.18	$N(\mu, \sigma^2)$	normal distribution with mean $\mu$ and variance $\sigma^2$
11.19	$Z \sim N(0,1)$	standard Normal distribution
11.20	$\phi$	probability density function of the standardised normal variable with distribution $N(0, 1)$
11.21	$\Phi$	corresponding cumulative distribution function
11.22	$\mu$	population mean
11.23	$\sigma^2$	population variance
11.24	$\sigma$	population standard deviation
11.25	$\bar{x}$	sample mean
11.26	$s^2$	sample variance
11.27	$s$	sample standard deviation
11.28	$H_0$	Null hypothesis,
11.29	$H_1$	Alternative hypothesis
11.30	$r$	product moment correlation coefficient for a sample

11.31	$\rho$	product moment correlation coefficient for a population
<b>12</b>	<b>Mechanics (Mathematics)</b>	
12.1	kg	kilograms
12.2	m	metres
12.3	km	kilometres
12.4	m/s, ms <sup>-1</sup>	metres per second (velocity)
12.5	m/s <sup>2</sup> , ms <sup>-2</sup>	metres per second per second (acceleration)
12.6	$F$	Force or resultant force
12.7	N	Newton
12.8	N m	Newton metre (moment of a force)
12.9	$t$	time
12.10	$s$	displacement
12.11	$u$	initial velocity
12.12	$v$	velocity or final velocity
12.13	$a$	acceleration
12.14	$g$	acceleration due to gravity
12.15	$\mu$	coefficient of friction



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## Mathematical Formulae that students are required to recall for AS and A Level Mathematics

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### Pure Mathematics

#### Quadratic Equations

$$ax^2 + bx + c = 0 \text{ has roots } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### Laws of Indices

$$a^x a^y \equiv a^{x+y}$$

$$a^x \div a^y \equiv a^{x-y}$$

$$(a^x)^y \equiv a^{xy}$$

#### Laws of Logarithms

$$x = a^n \Leftrightarrow n = \log_a x \text{ for } a > 0 \text{ and } x > 0$$

$$\log_a x + \log_a y \equiv \log_a (xy)$$

$$\log_a x - \log_a y \equiv \log_a \left( \frac{x}{y} \right)$$

$$k \log_a x \equiv \log_a (x^k)$$

#### Coordinate Geometry

A straight line graph, gradient  $m$  passing through  $(x_1, y_1)$  has equation

$$y - y_1 = m(x - x_1)$$

Straight lines with gradients  $m_1$  and  $m_2$  are perpendicular when  $m_1 m_2 = -1$

## Sequences

General term of an arithmetic progression:

$$u_n = a + (n - 1)d$$

General term of a geometric progression:

$$u_n = ar^{n-1}$$

## Trigonometry

In the triangle ABC

Sine rule:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule:  $a^2 = b^2 + c^2 - 2bc \cos A$

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$\cos^2 A + \sin^2 A \equiv 1$$

$$\sec^2 A \equiv 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$$

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A$$

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$$

## Mensuration

Area and Circumference of circle, radius  $r$  and diameter  $d$ :

$$C = 2\pi r = \pi d \quad A = \pi r^2$$

Pythagoras' Theorem: In any right-angled triangle where  $a$ ,  $b$  and  $c$  are the length of the sides and  $c$  is the hypotenuse:

$$c^2 = a^2 + b^2$$

Area of a trapezium =  $\frac{1}{2}(a + b)h$ , where  $a$  and  $b$  are the lengths of the parallel sides and  $h$  is their perpendicular separation.

Volume of a prism = area of cross section  $\times$  length

For a circle of radius  $r$ , where an angle at the centre of  $\theta$  radians subtends an arc of length  $l$  and encloses an associated sector of area  $a$ :

$$l = r\theta \quad a = \frac{1}{2}r^2\theta$$

## Calculus and Differential Equations

### Differentiation

Function	Derivative
$x^n$	$nx^{n-1}$
$\sin kx$	$k \cos kx$
$\cos kx$	$-k \sin kx$
$e^{kx}$	$ke^{kx}$
$\ln x$	$\frac{1}{x}$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

### Integration

Function	Integral
$x^n$	$\frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\cos kx$	$\frac{1}{k} \sin kx + c$
$\sin kx$	$-\frac{1}{k} \cos kx + c$
$e^{kx}$	$\frac{1}{k} e^{kx} + c$
$\frac{1}{x}$	$\ln x  + c, x \neq 0$
$f'(x) + g'(x)$	$f(x) + g(x) + c$
$f'(g(x))g'(x)$	$f(g(x)) + c$

$$\text{Area under a curve} = \int_a^b y \, dx \quad (y \geq 0)$$

## **Vectors**

$$|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{(x^2 + y^2 + z^2)}$$

## **Mechanics**

### **Forces and Equilibrium**

$$\text{Weight} = \text{mass} \times g$$

$$\text{Friction: } F \leq \mu R$$

Newton's second law in the form:  $F = ma$

### **Kinematics**

For motion in a straight line with variable acceleration:

$$v = \frac{dr}{dt} \quad a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$$

$$r = \int v dt \quad v = \int a dt$$

## **Statistics**

$$\text{The mean of a set of data: } \bar{x} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$$

$$\text{The standard Normal variable: } Z = \frac{X - \mu}{\sigma} \quad \text{where } X \sim N(\mu, \sigma^2)$$

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$$l = r\theta \quad a = \frac{1}{2}r^2\theta$$

### Complex Numbers

For two complex numbers  $z_1 = r_1e^{i\theta_1}$  and  $z_2 = r_2e^{i\theta_2}$ :

$$z_1z_2 = r_1r_2e^{i(\theta_1+\theta_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}e^{i(\theta_1-\theta_2)}$$

Loci in the Argand diagram:

$|z - a| = r$  is a circle radius  $r$  centred at  $a$

$\arg(z - a) = \theta$  is a half line drawn from  $a$  at angle  $\theta$  to a line parallel to the positive real axis

Exponential Form:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

### Matrices

For a 2 by 2 matrix  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  the determinant  $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

the inverse is  $\frac{1}{\Delta} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

The transformation represented by matrix **AB** is the transformation represented by matrix **B** followed by the transformation represented by matrix **A**.

For matrices **A**, **B**:

$$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$

## Algebra

$$\sum_{r=1}^n r = \frac{1}{2}n(n+1)$$

For  $ax^2 + bx + c = 0$  with roots  $\alpha$  and  $\beta$  :

$$\alpha + \beta = \frac{-b}{a} \quad \alpha\beta = \frac{c}{a}$$

For  $ax^3 + bx^2 + cx + d = 0$  with roots  $\alpha$ ,  $\beta$  and  $\gamma$  :

$$\sum \alpha = \frac{-b}{a} \quad \sum \alpha\beta = \frac{c}{a} \quad \alpha\beta\gamma = \frac{-d}{a}$$

## Hyperbolic Functions

$$\cosh x \equiv \frac{1}{2}(e^x + e^{-x})$$

$$\sinh x \equiv \frac{1}{2}(e^x - e^{-x})$$

$$\tanh x \equiv \frac{\sinh x}{\cosh x}$$

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$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$



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Function	Integral
$x^n$	$\frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\cos kx$	$\frac{1}{k} \sin kx + c$
$\sin kx$	$-\frac{1}{k} \cos kx + c$
$\cosh kx$	$\frac{1}{k} \sinh kx + c$
$\sinh kx$	$\frac{1}{k} \cosh kx + c$
$e^{kx}$	$\frac{1}{k} e^{kx} + c$
$\frac{1}{x}$	$\ln x  + c, x \neq 0$
$f'(x) + g'(x)$	$f(x) + g(x) + c$
$f'(g(x))g'(x)$	$f(g(x)) + c$

Area under a curve  $= \int_a^b y \, dx$  ( $y \geq 0$ )

Volumes of revolution about the  $x$  and  $y$  axes:

$$V_x = \pi \int_a^b y^2 \, dx \qquad V_y = \pi \int_c^d x^2 \, dy$$

Simple Harmonic Motion:

$$\ddot{x} = -\omega^2 x$$

## Vectors

$$|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{x^2 + y^2 + z^2}$$

Scalar product of two vectors  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$  is

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta$$

where  $\theta$  is the acute angle between the vectors  $\mathbf{a}$  and  $\mathbf{b}$

The equation of the line through the point with position vector  $\mathbf{a}$  parallel to vector  $\mathbf{b}$  is:

$$\mathbf{r} = \mathbf{a} + t\mathbf{b}$$

The equation of the plane containing the point with position vector  $\mathbf{a}$  and perpendicular to vector  $\mathbf{n}$  is:

$$(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0$$

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### Forces and Equilibrium

$$\text{Weight} = \text{mass} \times g$$

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## Statistics

The mean of a set of data:  $\bar{x} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$

The standard Normal variable:  $Z = \frac{X - \mu}{\sigma}$  where  $X \sim N(\mu, \sigma^2)$

## **Appendix D: Ofqual's role, objectives and duties**

Our statutory objectives include the qualifications standards objective, which is to secure that the qualifications we regulate:

- (a) give a reliable indication of knowledge, skills and understanding; and
- (b) indicate:
  - (i) a consistent level of attainment (including over time) between comparable regulated qualifications; and
  - (ii) a consistent level of attainment (but not over time) between qualifications we regulate and comparable qualifications (including those awarded outside the UK) that we do not regulate.

We must therefore regulate so that qualifications properly differentiate between students who have demonstrated that they have the knowledge, skills and understanding required to attain the qualification and those who have not.

We also have a duty under the Apprenticeship, Skills, Children and Learning Act 2009 to have regard to the reasonable requirements of relevant students, including those with special educational needs and disabilities, of employers and of the higher education sector, and to aspects of government policy when so directed by the Secretary of State.

As a public body, we are subject to the public sector equality duty.<sup>30</sup> This duty requires us to have due regard to the need to:

- (a) eliminate discrimination, harassment, victimisation and any other conduct that is prohibited under the Equality Act 2010;
- (b) advance equality of opportunity between persons who share a relevant protected characteristic and persons who do not share it;
- (c) foster good relations between persons who share a relevant protected characteristic and persons who do not share it.

The exam boards that design, deliver and award AS and A level qualifications are required by the Equality Act, among other things, to make reasonable adjustments for disabled people taking their qualifications, except where we have specified that such adjustments should not be made.

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<sup>30</sup> Equality Act 2010, section 149.

When we decide whether such adjustments should not be made, we must have regard to:

- (a) the need to minimise the extent to which disabled persons are disadvantaged in attaining the qualification because of their disabilities;
- (b) the need to secure that the qualification gives a reliable indication of the knowledge, skills and understanding of a person upon whom it is conferred;
- (c) the need to maintain public confidence in the qualification.

Legislation therefore sets out a framework within which we must operate. We are subject to a number of duties and we must aim to achieve a number of objectives. These different duties and objectives can, from time to time, conflict with each other. For example, if we regulate to secure that a qualification gives a reliable indication of a student's knowledge, skills and understanding, a student who has not been able to demonstrate the required knowledge, skills and/or understanding will not be awarded the qualification. A person may find it more difficult, or impossible, to demonstrate the required knowledge, skills and/or understanding because they have a protected characteristic. This could put them at a disadvantage relative to others who have been awarded the qualification. It is not always possible for us to regulate so that we can both secure that qualifications give a reliable indication of knowledge, skills and understanding and advance equality between people who share a protected characteristic and those who do not. We must review all the available evidence and actively consider all the available options before coming to a final, rational decision.

Qualifications cannot be used to mitigate inequalities or unfairness in the education system or in society more widely than might affect, for example, students' preparedness to take the qualification and the assessments within it. While a wide range of factors can have an impact on a student's ability to achieve a particular mark in an assessment, our influence is limited to the way the qualification is designed and assessed.

We require the exam boards to design qualifications to give a reliable indication of the knowledge, skills and understanding of those on whom they are conferred. We also require the exam boards to avoid, where possible, features of a qualification that could, without justification, make a qualification more difficult for a student to achieve because they have a particular protected characteristic. We require exam boards to monitor whether any features of their qualifications have this effect.

In setting the overall framework within which exam boards will design, assess and award the reformed GCSE, A level and AS qualifications, we want to understand the possible impacts of the proposals on persons who share a protected characteristic.

The protected characteristics under the Equality Act 2010 are:

- age;
- disability;
- gender reassignment;
- marriage and civil partnerships;
- pregnancy and maternity;
- race;
- religion or belief;
- sex;
- sexual orientation.

It should be noted that with respect to the public sector equality duty under section 149 of the 2010 Act, we are not required to have due regard to impacts on those who are married or in a civil partnership.

## **Responding to the consultation**

### **Your details**

To evaluate responses properly, we need to know who is responding to the consultation and in what capacity. We will therefore only consider your response if you complete the following information section.

We will publish our evaluation of responses. Please note that we may publish all or part of your response unless you tell us (in your answer to the confidentiality question) that you want us to treat your response as confidential. If you tell us you wish your response to be treated as confidential, we will not include your details in any published list of respondents, although we may quote from your response anonymously.

Please answer all questions marked with a star\*

Name\*

Position\*

Organisation name (if applicable)\*

Address

Email

Telephone

**Would you like us to treat your response as confidential?\***

If you answer yes, we will not include your details in any list of people or organisations that responded to the consultation.

Yes  No

**Is this a personal response or an official response on behalf of your organisation?\***

Personal response (please answer the question ‘If you ticked “Personal response”...’)

Official response (please answer the question ‘If you ticked “Official response”...’)

**If you ticked “Personal response”, which of the following are you?**

Student

Parent or carer

Teacher (but responding in a personal capacity)

Other, including general public (please state below)

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**If you ticked “Official response”, please respond accordingly:**

**Type of responding organisation\***

Awarding organisation

Local authority

School or college (please answer the question below)

Academy chain

Private training provider

University or other higher education institution

Employer

Other representative or interest group (please answer the question below)



**School or college type**

- Comprehensive or non-selective academy
  - State selective or selective academy
  - Independent
  - Special school
  - Further education college
  - Sixth form college
  - Other (please state below)
- 

**Type of representative group or interest group**

- Group of awarding organisations
  - Union
  - Employer or business representative group
  - Subject association or learned society
  - Equality organisation or group
  - School, college or teacher representative group
  - Other (please state below)
- 

**Nation\***

- England
- Wales
- Northern Ireland
- Scotland
- Other EU country: \_\_\_\_\_
- Non-EU country: \_\_\_\_\_

**How did you find out about this consultation?**

Our newsletter or another one of our communications

Our website

Internet search

Other

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**May we contact you for further information?**

Yes  No

## Questions

**Question 1: To what extent do you agree or disagree that the proposed assessment objectives are appropriate for AS and A level mathematics and further mathematics?**

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree

Please explain your reasons:

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.....

**Question 2: To what extent do you agree or disagree that the proposed weightings of the assessment objectives are appropriate for AS mathematics?**

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree

Please explain your reasons:

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**Question 3: To what extent do you agree or disagree that the proposed weightings of the assessment objectives are appropriate for A level mathematics?**

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree

Please explain your reasons:

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**Question 4: To what extent do you agree or disagree that the proposed weightings of the assessment objectives are appropriate for AS further mathematics?**

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree

Please explain your reasons:

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**Question 5: To what extent do you agree or disagree that the proposed weightings of the assessment objectives are appropriate for A level further mathematics?**

- Strongly agree
- Agree
- Neither agree nor disagree

Disagree

Strongly disagree

Please explain your reasons:

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**Question 6: To what extent do you agree or disagree that we should introduce a Condition which requires exam boards to comply with the relevant subject content – including the two proposed new appendices – and assessment objectives?**

Strongly agree

Agree

Neither agree nor disagree

Disagree

Strongly disagree

Please explain your reasons:

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.....  
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**Question 7: To what extent do you agree or disagree that we should introduce guidance which clarifies that awarding organisations should explain and justify in their assessment strategies how their qualification design reflects the ‘Overarching themes’ and ‘Use of technology’ sections of the subject content?**

Strongly agree

Agree

Neither agree nor disagree

Disagree

Strongly disagree

Please explain your reasons:

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**Question 8: To what extent do you agree or disagree that we should introduce guidance which clarifies how awarding organisations should interpret our assessment objectives?**

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree

Please explain your reasons:

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**Question 9: To what extent do you agree or disagree that we should allow the first exams for new mathematics A levels in summer 2018 (at the end of the first year of teaching)?**

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree

Please explain your reasons:

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**Question 10: To what extent do you agree or disagree with our proposed approach to regulating the sampling of subject content in AS and A level mathematics?**

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree

Please explain your reasons:

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**Question 11: To what extent do you agree or disagree with our proposed approach to regulating the use and assessment of large data sets in AS and A level mathematics?**

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree

Please explain your reasons:

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**Question 12: To what extent do you agree or disagree with our proposed approach to regulating non-core content in AS and A level further mathematics?**

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree

Please explain your reasons:

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**Question 13: Do you have any comments on our proposed Conditions and requirements for AS and A level mathematics?**

- Yes  No

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**Question 14: Do you have any comments on our proposed Conditions and requirements for AS and A level further mathematics?**

- Yes  No

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**Question 15: Do you have any comments on our proposed guidance for AS and A level mathematics?**

- Yes  No

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**Question 16: Do you have any comments on our proposed guidance for AS and A level further mathematics?**

Yes  No

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**Question 17: Do you have any comments on DfE’s proposed new appendices to the subject content for mathematics and further mathematics?**

Yes  No

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**Question 18: We have not identified any ways in which the proposals for AS and A level mathematics and further mathematics would impact (positively or negatively) on persons who share a protected characteristic.<sup>31</sup> Are there any potential impacts we have not identified?**

Yes  No

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**Question 19: Are there any additional steps we could take to mitigate any negative impact resulting from these proposals on persons who share a protected characteristic?**

Yes  No

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<sup>31</sup> ‘Protected characteristic’ is defined in the Equality Act 2010. Here, it means disability, racial group, age, religion or belief, pregnancy or maternity, sex, sexual orientation and gender reassignment.

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**Question 20: Have you any other comments on the impacts of the proposals on students who share a protected characteristic?**

Yes  No

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## Accessibility of our consultations

We are looking at how we provide accessible versions of our consultations and would appreciate it if you could spare a few moments to answer the following questions. Your answers to these questions will not be considered as part of the consultation and will not be released to any third parties.

**We want to write clearly, directly and put the reader first. Overall, do you think we have got this right in this consultation?**

Yes  No

**Do you have any comments or suggestions about the style of writing?**

Yes  No

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**Do you have any special requirements to enable you to read our consultations? (for example, screen reader, large text, and so on)**

Yes  No

**Which of the following do you currently use to access our consultation documents? (select all that apply)**

- Screen reader / text-to-speech software
- Braille reader
- Screen magnifier
- Speech-to-text software
- Motor assistance (blow-suck tube, mouth stick, and so on)
- Other .....

**Which of the following document formats would meet your needs for accessing our consultations? (select all that apply)**

- A standard PDF
- Accessible web pages
- Large-type PDF (16 point text)
- Large-type Word document (16 point text)
- eBook (Kindle, iBooks, or similar format)
- Braille document
- Spoken document
- Other .....

**How many of our consultations have you read in the last 12 months?**

- 1
- 2
- 3
- 4
- 5
- More than 5

We wish to make our publications widely accessible. Please contact us at [publications@ofqual.gov.uk](mailto:publications@ofqual.gov.uk) if you have any specific accessibility requirements.



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