

Defra / Environment Agency Flood and Coastal Defence R&D Programme



Benchmarking Hydraulic River Modelling Software Packages

Results – Test A (Subcritical, Supercritical & Transitional Flows)

R&D Technical Report: W5-105/TR2A

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Research Contractor: Bullen Consultants

EXECUTIVE SUMMARY

The test has successfully benchmarked and assessed the capability of ISIS, MIKE 11 and HEC-RAS with respect to modelling subcritical, supercritical and transitional flows. It has been found that all three software packages are capable of modelling subcritical, supercritical and transitional flows to an acceptable order of accuracy for the given parts of the test. However, use of the default calculation settings and calculation options may not be appropriate.

When a water surface profile is required in regions of transitional flow i.e. when flow changes from subcritical to supercritical flow, then ISIS (transcritical solver option) and HEC-RAS (with appropriate use of expansion and contraction coefficients) calculates a water surface profile through the transition that is close to the analytical solution. Conversely, MIKE 11 will smooth out the water surface profile. During a quasi-steady, and by inference unsteady simulation, the water surface profile in such regions is smooth as calculated by all three software packages.

When undertaking a steady state simulation with ISIS that has a supercritical flow component, then the transcritical solver option provides the most accurate solution when comparing results with the analytical solution. Hence, a recommendation of this study is to use the transcritical solver when appropriate.

Use of the default expansion and contraction coefficients in HEC-RAS can lead to oscillations in the water surface profile if inappropriately used. The default values of 0.1 and 0.3, for the contraction and expansion coefficients, are for subcritical flow only. For areas of supercritical flow, values of the order of 0.05 and 0.1 should be used where there are channel transitions. If there are no channel transitions, the coefficients should be set to zero. Furthermore, the contraction and expansion coefficients are only used in the solution of the energy equation (i.e. steady flow). They are not used in the solution of the momentum equation (quasi-steady/unsteady flow).

When undertaking quasi-steady, and by analogy unsteady simulations, HEC-RAS has produced the smallest overall RMS error (0.05) from the test configurations studied (when considering the most appropriate settings). However, this is only marginal when compared to ISIS and MIKE 11, which have both produced an RMS error of 0.06.

The hydraulic radius option in MIKE 11 produces a water surface profile that is akin to those determined by both ISIS and HEC-RAS. However, when using the default resistance radius option, which has been developed for use with natural channels and not rectangular channels as defined by the test, the result is generally inferior.

For parts 2 to 5 of the test, each of the software packages consistently underestimate the water level at the upstream boundary when undertaking a quasi-steady calculation.

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1 BACKGROUND

This report presents the results and findings from Test A (Subcritical and Supercritical Flows) of the Environment Agency of England and Wales (EA), Benchmarking and Scoping Study (2004). The study, which encompasses a series of tests, is intended to be an independent research investigation into the accuracy, capability and suitability of the following one-dimensional hydraulic river modelling software packages:

Software	Version	Developer	
ISIS	User Interface:	2.0 (13/01/01)	Halcrow /
	Flow Engine:	5.0.1 (27/06/01)	Wallingford Software
MIKE11	User Interface:	Build 5-052 (2001b)	DHI Water and Environment
	Flow Engine:	5.0.5.5	
HEC-RAS	User Interface:	3.1.0 (Beta) (03/02)	US Corps of Engineers
	Pre-processor:	3.1.0 (Beta) (03/02)	
	Steady Flow Engine:	3.1.0 (Beta) (03/02)	
	Unsteady Flow Engine:	3.1.0 (Beta) (03/02)	
	Post-processor:	3.1.0 (Beta) (03/02)	

Each of the above software packages was tested in the previously undertaken benchmarking study (Crowder, 1997). They are currently on the EA's BIS-A list of software packages for one-dimensional hydraulic river modelling.

The test has been undertaken on behalf of the EA by the following team in accordance with the Benchmarking Test Specification: Test A – Subcritical and Supercritical Flows (Crowder *et al*, 2004).

	Role	Affiliation
Mr Andrew Pepper	EA Project Manager	ATPEC River Engineering
Dr Richard Crowder	Study Project Manager/ Tester	Bullen Consultants Ltd
Dr Nigel Wright	Advisor	University of Nottingham
Dr Chris Whitlow	Advisor	Eden Vale Modelling Services
Dr Andrew Sleight	Advisor	University of Leeds
Dr Chris Tomlin	Advisor	Environment Agency
Miss Josephine Allan	Tester	University of Leeds

1.1 Aim of Test

The aim of the test is to:

- assess the ability of each software package to calculate subcritical, supercritical and transitional flows;
- assess the numerical accuracy of the software packages with reference to analytical results, as derived by MacDonald, where appropriate; and

- present the particulars for developing and undertaking the tests (Model Build) with each of the software packages and the associated results so that others can repeat the test with their own software.

2 MODEL BUILD

2.1 Test Configuration

The test has been undertaken in accordance with the Benchmarking Test Specification - Test A2 (Crowder *et al*, 2004).

The test has been undertaken in six separate parts as defined below in Table 2.1.

Table 2.1 TEST A – Definition of Parts 1 to 6 of Test A

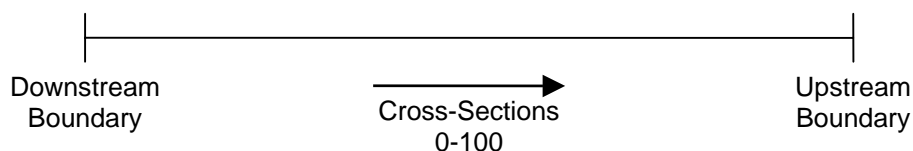
	Steady State	Quasi Steady	Unsteady
Part 1: Subcritical Flow	✓	✓	
Part 2: Supercritical Flow	✓	✓	
Part 3: Supercritical to Subcritical Flow	✓	✓	
Part 4: Subcritical to Supercritical to Subcritical Flow	✓	✓	
Part 5: Supercritical to Subcritical to Supercritical Flow	✓	✓	
Part 6: Transitional Flow			✓

For each part of the test 101 cross-sections of rectangular shape (10.0m wide) have been defined at 1.0m spacing with the bed level, as defined by the dataset specification.

The test configuration for all parts of the test is illustrated schematically in Figure 2.1.

A constant Manning's n value of 0.03 has been used throughout the reach for all parts of the test.

Figure 2.1 TEST A - Schematic Illustration of Test Configuration



For parts 1 to 5 of the test the software packages have been tested with separate steady state flow and water level boundary conditions, as defined in Table 2.2. In the supercritical runs, it should be noted that while a downstream boundary level should not be necessary needed for the calculations, a software package may require a downstream boundary as an input, even if it is not then used.

It is noted that it may be more appropriate to use the term ‘forewater’ instead of ‘backwater’ when referring to supercritical flows, however, for simplicity the term ‘backwater’ has been used throughout.

Table 2.2 TEST A – Boundary Conditions (Parts 1 to 5)

	Downstream Level (mAD)	Upstream Discharge (m ³ /s)
Part 1: Subcritical Flow	0.87803	20
Part 2: Supercritical Flow	0.67341	20
Part 3: Supercritical to Subcritical to Flow	0.61801	20
Part 4: Subcritical to Supercritical to Subcritical Flow	2.87904	20
Part 5: Supercritical to Subcritical to Supercritical Flow	0.61801	20

For parts 1 to 5 the software packages have also been tested under quasi-steady flow boundary conditions (i.e. an unsteady computation with constant boundary conditions). The steady state boundary conditions, as defined for each part of the test, were used at time $t = 0$ and extended through to 01:00hrs respectively. The data time interval was set at 1.0hr for each software package.

For part 6 of the test the upstream boundary was fixed at 20.0m³/s for a period of 24.0hrs. The downstream boundary was set at 0.602m between 00:00hrs and 06:00hrs after which it was linearly increased between 06:00hrs and 18:00hrs to 2.0m. It then was fixed at this level through to 24:00hrs.

2.2 Building the Model in ISIS, MIKE 11 and HEC-RAS

The model build with ISIS, MIKE 11 and HEC-RAS was undertaken in accordance with the test specification as defined by the dataset, with the following exceptions:

- ISIS and MIKE 11 could only handle three decimal places for the cross-section data and not the five decimal places as specified by the dataset; and
- ISIS and MIKE 11 could only handle three decimal places for the boundary data and not the five decimal places as specified by the dataset.

For most engineering purposes two decimal places is more than appropriate. As such the limitation of the ISIS and MIKE 11 software packages to three decimal places is not considered to be important as the fourth decimal place typically represents less than 0.1% error in the water depth in each part of the test.

The hydraulic reference manual for HEC-RAS (January 2001) p3-20, states that the default values of 0.1 and 0.3, for the contraction and expansion coefficients, are for subcritical flow only. It also states that for areas of supercritical flow, values on the order of 0.05 and 0.1 should be used where there are channel transitions. If there are no channel transitions, the

coefficients should be set to zero. Hence, the test was repeated with the contraction and expansion value set to 0.0.

When constructing the model in MIKE 11 the default “Resistance Radius” option in the cross-section editor was initially used. However, the test was also set up with the “Hydraulic Radius (Effective Area)” option so as to enable a comparison of the results from the two options.

The Resistance Radius formulation has been developed for use with natural channels, especially those incorporating floodplain sections. In such cases this formulation is designed to ensure a smooth increase in the section conveyance, which the hydraulic radius does not. However, for prismatic or steep sided channels, the resistance radius formulation may generate a section conveyance which is not consistent with user’s expectations of the Manning ‘n’ for the channel (which is based on the hydraulic radius, A/P). In these cases it is recommended by the developers that the user should select the hydraulic radius formulation. The default formulation can be changed in the cross section editor, under Settings/Miscellaneous. In addition, it is possible to switch formulations for any cross-section and recompute the processed data. Full details on these formulations are provided in the MIKE 11 user manuals in pdf.

3 RUNNING THE MODEL

3.1 Running the Model in ISIS

ISIS was first run in steady state mode for parts 1 to 5 of the test with the default run options. It should be noted that this employs the “direct method” solution method.

The diagnostics file (zzd) for the steady state runs indicated no errors for parts 1 to 5. However, for parts 2 to 5 a number of warnings were provided, which can be summarised as follows:

- Part 2 - The simplified method was used to compute the solution at chainages 0.0m through to 100.0m.
- Part 3 - The simplified method was used to compute the solution at chainages 0.0m through to 80.0m.
- Part 4 - Extra sections added by the direct method at chainages 35.0m and 36.0m. The simplified method was used to compute the solution at chainages 35.0m through to 80.0m.
- Part 5 - The simplified method was used to compute the solution at chainages 0.0m through to 100.0m.

The simplified method is used in ISIS where supercritical flow occurs. The dA/dx part of the convective momentum term in the momentum equation, when the Froude number exceeds a specified upper value, is neglected. Between this upper value and a specified lower value, the term is gradually phased out so that a smooth transition is achieved.

As an alternative to the default run option (direct method) the “direct method transcritical solver” option was also run as a separate test for parts 2 to 5. It should be noted that this option is only available for a steady run and is greyed out if performing an unsteady calculation.

The transcritical solver uses the full St Venant equations, with the numerical scheme reversing the direction of integration to upstream-to-downstream in supercritical parts of reaches. Momentum considerations are then used to establish where the supercritical and subcritical regimes meet, thus determining the locations of hydraulic jumps.

For parts 1 to 5 of the quasi-steady simulations, and also part 6 with an unsteady simulation, a time step of 20s was used. The results from the respective steady state simulations were used as initial conditions on each occasion.

The diagnostics files (zzd) for the quasi-steady runs again indicated no errors for parts 1 to 5, however, for parts 1 to 5 a number of warnings were provided, which can be summarised as follows:

- Part 1 - The simplified method was used to compute the solution at the following chainages: 100.0m, 99.0m, 98.0m, 97.0m, 96.0m, 95.0, 4.0m, 3.0m, 2.0m, 1.0m and 0.0m.
- Part 2 - The simplified method was used to compute the solution at chainages 0.0m through to 100.0m.

- Part 3 - The simplified method was used to compute the solution at chainages 0.0m through to 94.0m.
- Part 4 - The simplified method was used to compute the solution at chainages 44.0m through to 91.0m.
- Part 5 - The simplified method was used to compute the solution at chainages 0.0m through to 100.0m.

The same warning as provided for part 5 of the test was also provided for part 6 of the test (unsteady simulation).

3.2 Running the Model in MIKE 11

MIKE 11 does not provide the facility to undertake a steady state calculation, hence as a workaround MIKE 11 was run in unsteady mode with the quasi-steady boundary conditions and the type of initial condition set as steady state. This forced MIKE 11 to undertake a steady state calculation at $t = 0s$ and use the steady state result as an initial condition for the quasi-steady simulation. The result at $t = 0s$ was then taken as the steady state solution. For all parts of the test a time step of 20s was used.

The types of initial condition available with MIKE 11 are:

- **Steady State:** Initial conditions are calculated automatically assuming a steady state condition with discharges and water levels at the boundaries corresponding to the start time of the simulation.
- **Parameter File:** Initial conditions are taken from the parameter file relevant to the module in question.
- **Hotstart:** Initial conditions are loaded from an existing result file.
- **Steady+ Parameter:** Initial conditions are established using both the steady state and parameter file method.

For parts 1 to 5 of the quasi-steady simulations, and also part 6 with an unsteady simulation, a time step of 20s was used. The results from the respective steady state simulations were used as initial conditions on each occasion.

When running MIKE 11 with the default values for the computational scheme the results, for each part of the test, showed significant instabilities. However, by increasing the Delta value (the implicit weighting of the solution) from 0.50 to 0.55 in the default values for the computational scheme the instabilities were removed. Hence, a value of 0.55 for Delta was used for each part of the test so as to provide a satisfactory outcome.

In comparison, ISIS and HEC-RAS use equivalent default values for the computation scheme of 0.70 and 1.00 respectively.

In addition to undertaking the test with the default “Resistance Radius” resistance factor (in the cross-section editor) parts 1 to 5 of the test have been repeated using the alternative “Hydraulic Radius (Effective Area)” option.

MIKE 11 has two options for suppression of the convective acceleration term for flows approaching supercritical. According to the developers of MIKE 11... “the default, and most

stable option, gradually phases out this term at Froude numbers less than 1.0 with full suppression at Froude numbers of 1.0 and higher. A second, more advanced formulation does not alter the convective term until the Froude number exceeds 1.0. This formulation has the disadvantage that it is slightly less stable than the default". A full explanation is provided in Section 1.32 of the MIKE 11 Reference Manual. It has been beyond the scope of this study to investigate and to assess the advanced formulation.

3.3 Running the Model in HEC-RAS

HEC-RAS was first run in steady state mode for parts 1 to 5 of the test with the default run options.

When undertaking the steady state calculation for the parts of the test that had supercritical flow (i.e. parts 2 to 5) an upstream boundary had to be specified. This is a standard procedure in HEC-RAS. The value used in each instance was the respective analytical water level. An alternative approach could have been to use the critical depth boundary option within HEC-RAS.

The diagnostics file for the steady state runs provided no warnings for parts 1 and 2. However, for parts 3, 4 and 5 a number of warnings were provided, which can be summarised as follows:

Part 3 - At chainages 14.0m, 29.0m and 37.0m: The energy equation could not be balanced within the specified number of iterations. The program selected the water surface that had the least amount of error between computed and assumed values.

At chainages 14.0m, 29.0m and 37.0m: During the standard step iterations, when the assumed water surface was set equal to the critical depth, the calculated water surface came back below critical depth. This indicates that there is not a valid subcritical answer. The program defaulted to critical depth.

Part 4 - At chainage 47.0m: The energy equation could not be balanced within the specified number of iterations. The program selected the water surface that had the least amount of error between computed and assumed values.

At chainage 47.0m: During the standard step iterations, when the assumed water surface was set equal to the critical depth, the calculated water surface came back below critical depth. This indicates that there is not a valid subcritical answer. The program defaulted to critical depth.

Part 5 - At chainage 22.0m, 35.0m and 92.0m: The energy equation could not be balanced within the specified number of iterations. The program selected the water surface that had the least amount of error between computed and assumed values.

At chainage 22.0m, 35.0m, 92.0m: During the standard step iterations, when the assumed water surface was set equal to the critical depth, the calculated water surface came back below critical depth. This indicates that there is not a

valid subcritical answer. The program defaulted to critical depth.

For parts 1 to 5 of the quasi-steady simulations, and also part 6 with an unsteady simulation, a time step of 20s was used. In each instance initial conditions were automatically generated by HEC-RAS (undertaking a steady state calculation) based on an initial flow of $20.0\text{m}^3/\text{s}$ and the quasi-steady boundary water levels that were defined at $t = 0\text{s}$.

The diagnostics file for the quasi-steady and unsteady runs were checked and indicated no warnings for each part of the test.

As discussed in section 2.2, the expansion and contraction values for rectangular channels should be set to zero to avoid the production of oscillations in the water surface profile during supercritical flow. Hence, to enable an assessment of their influence on the results, parts 1 to 5 of the test were undertaken with the default and recommended expansion and contraction values. Part 6 was not tested for this as these values are not used in an unsteady calculation.

When running HEC-RAS with the contraction and expansion coefficients set to zero, the following warnings were removed:

- Part 3: Both warnings removed at chainage 14.0m and 29.0m.
- Part 5: Both warnings removed at chainage 22.0m and 92.0m.

4 RESULTS

4.1 Introduction

For each part of the test the results from all the software packages have been discussed, compared and presented in combination so as to provide a direct comparison.

For ISIS, additional analysis has been undertaken for parts 2 to 5, which compares the results of the standard solver with the transcritical solver.

For HEC-RAS, additional analysis has been undertaken for parts 1 to 5, which compares results when using the default cross sections contraction and expansion coefficients of 0.1 and 0.3 respectively and alternative values of 0.0 for both.

For MIKE 11, additional analysis has been undertaken for parts 1 to 5, which compares results when using the default “Resistance Radius” cross section option with the alternative “Hydraulic Radius (Effective Area)” option.

The analysis of results for parts 1 to 5 of the test has been limited to the following for both the steady state and quasi-steady simulations:

- Longitudinal water surface profile, with comparison to analytical solution;
- Longitudinal profile showing the percentage difference between analytical and numerical solution for longitudinal water surface profile; and
- Root Mean Square (RMS) Error over the length of the reach for each part of the test.

The analysis of results for part 6 of the test has been limited to the following:

- Stage verses Time at 20.0m, 40.0m, 60.0m and 80.0m from the downstream boundary.

The water level results and percentage difference between the analytical and numerical solutions for each software package for parts 1 to 5 of the test are illustrated in Appendix A, Graphs 1 to 68. The water level profile results for each software package for part 6 of the test are illustrated in Appendix A, Graphs 69 to 72.

The Root Mean Square Errors associated with the numerical results for each software package for parts 1 to 5 of the test are illustrated in Figure 4.1 and presented in Table 4.1.

The solutions with the smallest RMS error for each of the software packages from the simulations undertaken are illustrated in Figure 4.2 for parts 1 to 5.

Figure 4.1 Root Mean Square Errors Test A Parts 1 to 5

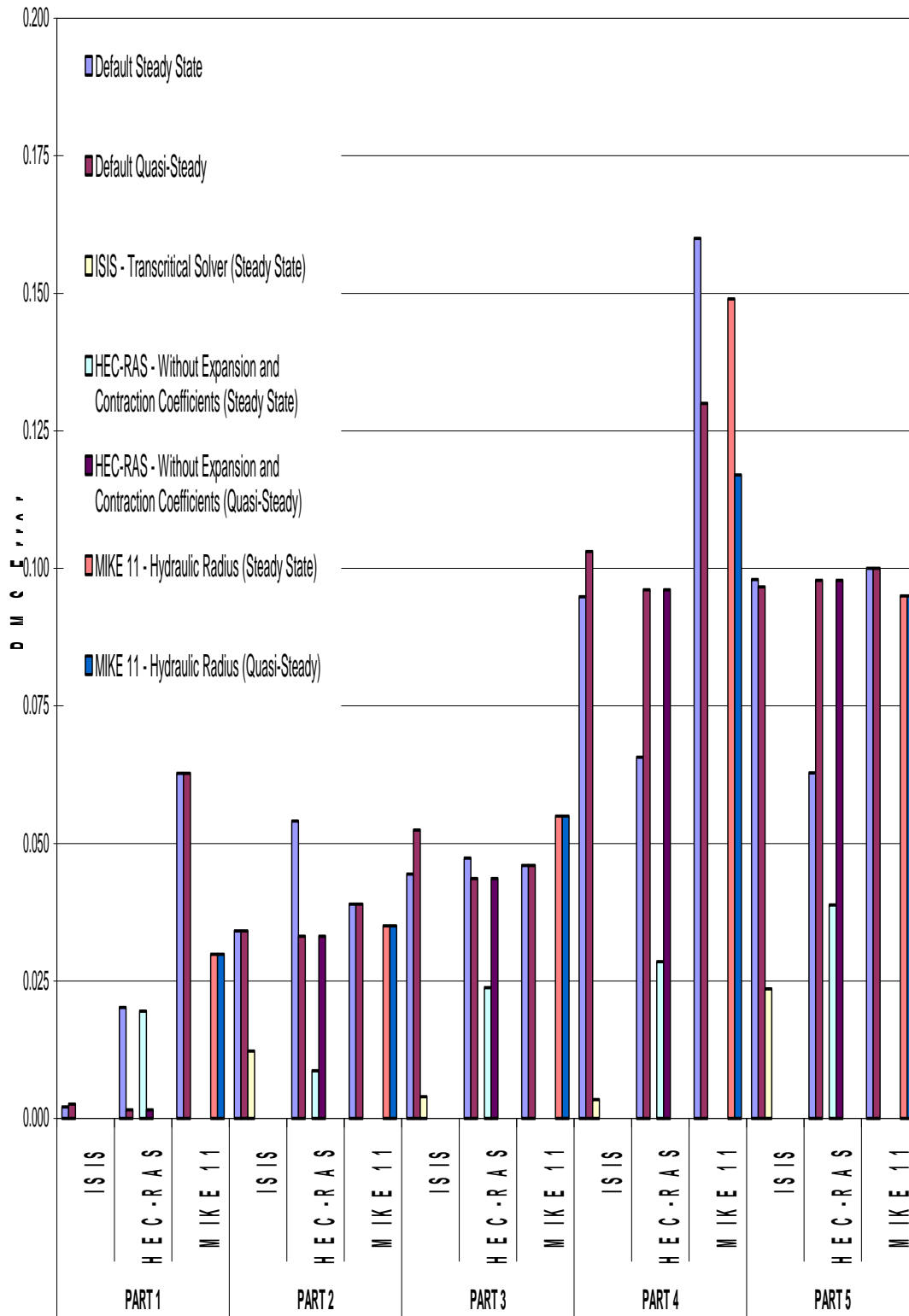


Table 4.1 TEST A – Root Mean Square Errors (Parts 1 to 5)

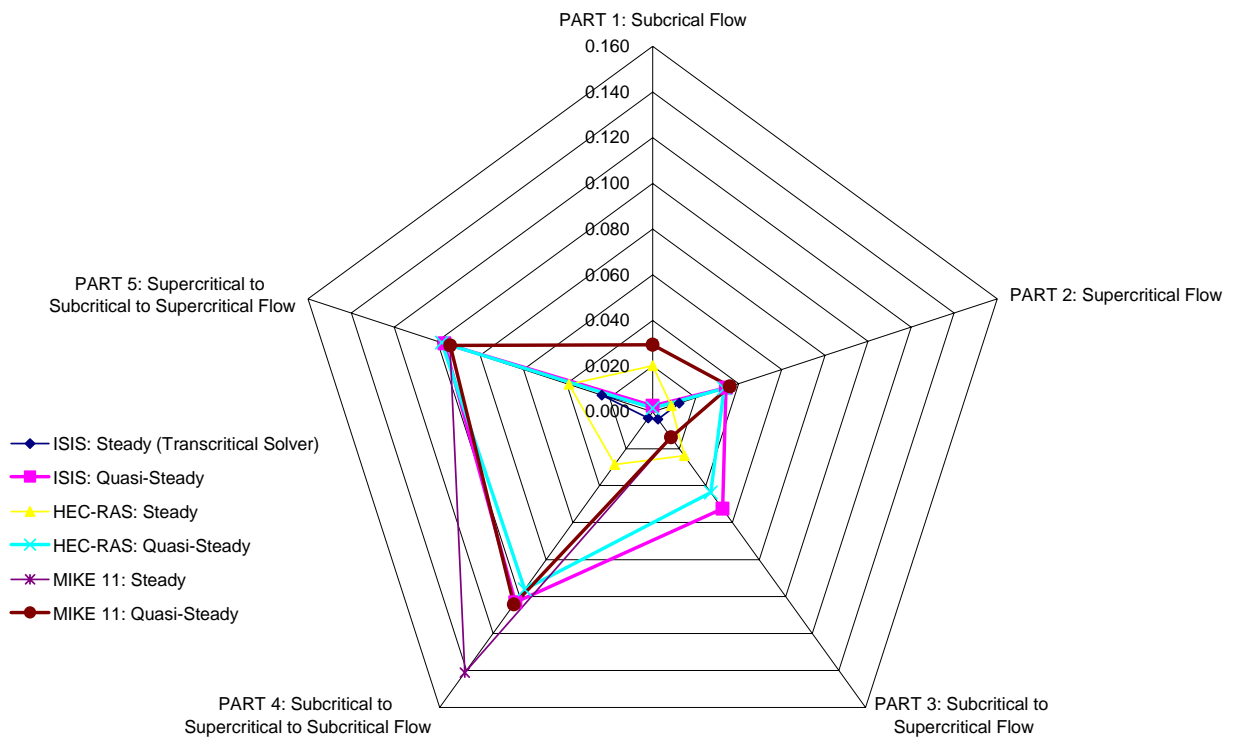
		Default Steady State	Default Quasi-Steady	ISIS - Transcritical Solver (Steady State)	HEC-RAS - Without Expansion and Contraction Coefficients (Steady State)	HEC-RAS - Without Expansion and Contraction Coefficients (Quasi-Steady)	MIKE 11 - Hydraulic Radius (Steady State)	MIKE 11 - Hydraulic Radius (Quasi-Steady)
PART 1	ISIS	0.002	0.003					
	HEC-RAS	0.020	0.002		0.020	0.002		
	MIKE 11	0.063	0.063				0.030	0.030
PART 2	ISIS	0.034	0.034	0.012				
	HEC-RAS	0.054	0.033		0.009	0.033		
	MIKE 11	0.039	0.039				0.035	0.035
PART 3	ISIS	0.044	0.052	0.004				
	HEC-RAS	0.047	0.044		0.024	0.044		
	MIKE 11	0.046	0.046				0.055	0.055
PART 4	ISIS	0.095	0.103	0.003				
	HEC-RAS	0.066	0.096		0.028	0.096		
	MIKE 11	0.160	0.130				0.149	0.117
PART 5	ISIS	0.098	0.097	0.024				
	HEC-RAS	0.063	0.098		0.039	0.098		
	MIKE 11	0.100	0.100				0.095	0.095

The average of the RMS errors for each of the software packages for both the steady and quasi-steady tests is given in Table 4.2.

Table 4.2 TEST A – Average RMS Errors

	Average RMS	
	Steady	Quasi-Steady
ISIS	0.01	0.06
HEC-RAS	0.02	0.05
MIKE 11	0.07	0.06

Figure 4.2 TEST A – Smallest Root Mean Square Errors for each software package (Parts 1 to 5)



4.2 Analysis of Results - Part 1: Subcritical Flow

Inspection of Graph 1 shows that the water levels produced by each of the software packages for a simple steady state subcritical flow regime, with default program settings, are very smooth with no visible fluctuations. In each instance the predicted water level closely follows the analytical solution. The percentage error in water depth when compared to the analytical solution, as illustrated in Graph 2, is almost zero for ISIS along the complete length of the channel, and is generally within $\pm 5\%$ for both MIKE 11 and HEC-RAS. The RMS error of 0.002 for ISIS is clearly the smallest of the three software packages.

Inspection of Graph 3 shows that the water levels produced by each of the software packages for the quasi-steady subcritical flow regime, with default program settings, are also very smooth with no visible fluctuations. In each instance the predicted water surface profile closely follows the analytical solution. The percentage error in water depth when compared to the analytical solution, as illustrated in Graph 4, is again almost zero for ISIS along the complete length of the channel, and is now matched by the result from HEC-RAS. However, the result from MIKE 11 is unchanged. The RMS error for ISIS is increased to 0.003, as compared to the steady state solution, whereas the RMS error for HEC-RAS is now 0.002 as compared to 0.020.

Using a value of 0.0 for the cross section contraction and expansion coefficients in HEC-RAS for either the steady state or quasi-steady simulation does not change the respective results in any way, as illustrated in Graphs 41 to 44, Figure 4.1 and Table 4.1.

Using the cross section hydraulic radius option in MIKE 11, the RMS error is reduced from 0.063 to 0.030 for both the steady and quasi-steady runs, as illustrated Figure 4.1 and Table 4.1. The water level and percentage errors being illustrated in Graphs 21 to 24.

4.3 Analysis of Results - Part 2: Supercritical Flow

Inspection of Graph 5 shows the differences in the water levels produced by each of the software packages for the steady state supercritical flow regime, with default program settings. For ISIS and MIKE 11 the water levels are very smooth with no visible fluctuations, however, for HEC-RAS there are clearly areas of varying water levels. All packages closely follow the analytical water surface profile.

Over the lower 45-53% of the channel ISIS and MIKE 11 over predict the water level by up to 3.8% and 7.6% respectively and for the upper both under predict the water level by up to 10.7% and 7.4% respectively. Inspection of Graph 6 shows that the percentage error follows an approximation to a sinusoidal profile. The results from HEC-RAS are somewhat different in profile and magnitude of error.

For the steady state calculations both ISIS and MIKE 11 have similar RMS errors of 0.034 and 0.039 whereas HEC-RAS has a RMS error of 0.054.

Inspection of Graph 7 shows that the water levels produced by each of the software packages for the quasi-steady supercritical flow regime, with default program settings, are also very smooth with no visible fluctuations. In each instance the predicted water level closely follows the analytical solution. The percentage error in water depth for ISIS and MIKE 11, as illustrated in Graph 8, is almost identical to that observed for the steady state solution.

However, the HEC-RAS percentage error is significantly reduced and is now almost identical to that of ISIS.

The RMS error for ISIS and MIKE 11 is unchanged at 0.034 and 0.036 respectively; however, HEC-RAS is reduced to 0.033 as compared to a value of 0.054 for the steady state simulation.

Using the transcritical solver in ISIS, the water level is almost identical to the analytical solution, as illustrated in Graph 61. The percentage error is close to zero along the complete length of the channel, as illustrated in Graph 62 and the RMS error is significantly reduced from 0.034 to 0.012.

Using a value of 0.0 for the cross section contraction and expansion coefficients in HEC-RAS makes an improvement in the results for the steady state simulation, as illustrated in Graphs 45 and 46. The fluctuations in the water level are eliminated and the percentage error is reduced to close to zero along the complete length of the channel. The RMS error is reduced to 0.009, which is the lowest value for all of the software packages.

Using a value of 0.0 for the cross section contraction and expansion coefficients in HEC-RAS for the quasi-steady state simulation makes no change to the results in any way, as illustrated in Graphs 47 and 48. The RMS error remains unchanged at 0.033.

Using the cross section hydraulic radius option in MIKE 11 the RMS error is reduced from 0.039 to 0.035 for both the steady state and quasi-steady results, as illustrated in Figure 4.1 and Table 4.1. The water level and percentage errors being illustrated in Graphs 25 to 28.

4.4 Analysis of Results - Part 3: Supercritical to Subcritical Flow

Implicitly none of the packages are appropriate for modelling the hydraulic jump itself; however, with careful application they can give a reasonable result as illustrated by Graph 9. Graph 9 shows the differences in the water levels produced by each of the software packages for the steady state supercritical flow regime, with default program settings. For ISIS and MIKE 11 the water levels are very smooth with no visible fluctuations, however, for HEC-RAS there are clearly areas of varying water levels in the downstream supercritical part of the channel. All packages follow the analytical water surface profile.

Over the complete length of the channel both ISIS and MIKE 11 under predict the water level by up to approximately 8%. Inspection of Graph 10 shows that the percentage gradually increases up to about chainage 75.0m for both ISIS and MIKE 11 before a rapid reduction in the error is observed for ISIS and a small and gradual reduction for MIKE 11. The results from HEC-RAS are somewhat worse in profile and magnitude of error.

All three software packages have similar RMS errors; 0.044 for ISIS, 0.046 for MIKE 11 and 0.047 for HEC-RAS, as shown in Table 4.1 and illustrated in Figure 4.1.

Inspection of Graph 11 shows that the water levels produced by each of the software packages for the quasi-steady supercritical flow regime, with default program settings, are very smooth with no visible fluctuations. In each instance the predicted water level closely follows the analytical solution. The percentage error in water depth for ISIS and MIKE 11, as illustrated in Graph 12, is almost identical to that observed for the steady state solution

although the reduction in error for ISIS after about chainage 75.0m is not as pronounced. The HEC-RAS percentage error is much smaller and is now very similar to that produced by the ISIS steady state result.

The RMS error for ISIS increases from 0.044 to 0.052, whereas the MIKE 11 RMS is unchanged at 0.046. However, the HEC-RAS RMS error is slightly reduced to 0.044 when compared to a value of 0.047 for the steady state simulation, as shown in Table 4.1 and illustrated in Figure 4.1.

Using the transcritical solver in ISIS, the water level is almost identical to the analytical solution, as illustrated in Graph 63. The percentage error is close to zero along the complete length of the channel, as illustrated in Graph 62 and the RMS error is significantly reduced from 0.044 to 0.004.

Using a value of 0.0 for the cross section contraction and expansion coefficients in HEC-RAS for the steady state simulation makes an improvement in the results in the supercritical region, as illustrated in Graphs 49 and 50. The fluctuations in the water level are eliminated and the percentage error is reduced to within $\pm 5\%$ along the complete length of the channel. The RMS error is reduced from 0.047 to 0.024.

Using a value of 0.0 for the cross section contraction and expansion coefficients in HEC-RAS for the quasi-steady state simulation makes no change to the results in any way, as illustrated in Graphs 51 and 52. The RMS error remains unchanged at 0.044.

Using the cross section hydraulic radius option in MIKE 11 the RMS error is increased from 0.046 to 0.055 for both the steady state and quasi-steady results, as illustrated Figure 4.1 and Table 4.1. The water level and percentage errors being illustrated in Graphs 29 to 32.

4.5 Analysis of Results - Part 4: Subcritical to Supercritical to Subcritical

Inspection of Graph 13 shows the differences in the water levels produced by each of the software packages for the steady state supercritical flow regime, with default program settings. For ISIS and MIKE 11 the water levels are very smooth with no visible fluctuations, however, through the area of the hydraulic jump the water level is smoothed out by each of the software packages, more so by MIKE 11. For HEC-RAS there are clearly areas of varying water levels in the area just upstream and downstream of the hydraulic jump, however, the profile of the hydraulic jump, when compared to the analytical solution, is closely followed.

All packages show an acceptable correlation to the analytical water surface profile upstream and downstream of the hydraulic jump.

Downstream of the hydraulic jump ISIS produces a water level error close to zero; however, approaching the hydraulic jump ISIS overestimates the water level before underestimating the water level for the length upstream of the jump, as illustrated in Graph 13. This underestimation is approximately 15% between chainages 50.0m and 70.0m. Upstream of 70.0m the underestimation gradually reduces to approximately 3% at the upstream boundary, as illustrated in Graph 14.

Downstream of the hydraulic jump MIKE 11 gradually overestimates the water level as the hydraulic jump is approached. Upstream of the jump the water level is underestimated. This underestimation peaks at approximately 14% at chainage 73.0m and then gradually reduces to approximately 12% at the upstream boundary.

Downstream of the hydraulic jump HEC-RAS produces a water level error that is close to zero up to chainage 20.0m and then overestimates the water level as the hydraulic jump is approached. Upstream of the jump the water level is underestimated. This under estimation peaks at approximately 5% at chainage 65.0m and then gradually reduces to approximately 3% at the upstream boundary.

The RMS error for each software package for the steady state simulation is 0.095 for ISIS, 0.160 for MIKE 11 and 0.066 for HEC-RAS, as shown in Table 4.1 and illustrated in Figure 4.1.

Inspection of Graph 15 shows that the water levels produced by each of the software packages for the quasi-steady supercritical flow regime, with default program settings, are very similar, smooth and have no visible fluctuations. In each instance the predicted water level closely follows the analytical solution. The percentage error in water depth for ISIS, MIKE 11 and HEC-RAS are very similar, as illustrated in Graph 16.

In comparison to the steady state solution the quasi-steady RMS error for ISIS and HEC-RAS increases from 0.095 to 0.103 and 0.066 to 0.096 respectively, whereas the MIKE 11 RMS error is reduced from 0.147 to 0.110, as shown in Table 4.1 and illustrated in Figure 4.1.

Using the transcritical solver in ISIS, the water level is almost identical to the analytical solution, as illustrated in Graph 65. The percentage error is close to zero along the complete length of the channel, as illustrated in Graph 66 and the RMS error is significantly reduced from 0.095 to 0.003.

Using a value of 0.0 for the cross section contraction and expansion coefficients in HEC-RAS for the steady state simulation makes an improvement in the results in the region of the hydraulic jump, as illustrated in Graphs 53 and 54. The fluctuations in the water level are reduced, which in turn reduces the RMS error from 0.066 to 0.028.

Using a value of 0.0 for the cross section contraction and expansion coefficients in HEC-RAS for the quasi-steady state simulation makes no change to the results, as illustrated in Graphs 55 and 56. The RMS error remains unchanged at 0.096.

Using the cross section hydraulic radius option in MIKE 11 the RMS error is reduced from 0.160 to 0.149 for the steady state result and from 0.130 to 0.117 for the quasi-steady results, as illustrated Figure 4.1 and Table 4.1. The water level and percentage errors being illustrated in Graphs 33 to 36.

4.6 Analysis of Results - Part 5: Supercritical to Subcritical to Supercritical

Inspection of Graph 17 shows the differences in the water levels produced by each of the software packages for the steady state supercritical flow regime, with default program settings. For ISIS and MIKE 11 the water levels are very smooth with no visible fluctuations, however, through the area of the hydraulic jump the water level is smoothed out by each of

the software packages. For HEC-RAS there are areas of varying water levels in the length close to the boundaries i.e. where supercritical flow occurs, however, the profile of the hydraulic jump, when compared to the analytical solution, is closely followed. All packages show a good correlation to the analytical water surface profile.

Downstream of the hydraulic jump ISIS and MIKE 11 underestimate the water level. The underestimation increases in the upstream direction with the error peaking at approximately 10% and 14% respectively, as illustrated by Graph 18. At the hydraulic jump both ISIS and MIKE 11 over estimate the water level before again rapidly underestimating the water level for the length upstream of the jump. For both ISIS and MIKE 11 this under estimation is approximately 16% and 19% respectively at the upstream boundary, as illustrated in Graph 18.

Downstream of the hydraulic jump HEC-RAS overestimates the water level by approximately 15%. Notably downstream of the hydraulic jump the water level significantly oscillates. As the hydraulic jump is approached these oscillations disappear at around chainage 35.0m and the water level gradually becomes underestimated by approximately 4%. At the hydraulic jump HEC-RAS suddenly overestimates the water level by approximately 10%. This rapidly reduces to around 1% at chainage 75.0m before gradually increasing with the development further oscillations, as illustrated in Graph 18.

The RMS error for the steady state simulation is 0.098 for ISIS, 0.010 for MIKE 11 and 0.063 for HEC-RAS, as shown in Table 4.1 and illustrated in Figure 4.1.

Inspection of Graph 19 shows that the water levels produced by each of the software packages for the quasi-steady supercritical flow regime, with default program settings, are very similar, smooth and have no visible fluctuations. In each instance the predicted water level closely follows the analytical solution; however, the water level is smoothed in the region of the hydraulic jump. The percentage error in water depth for ISIS, and HEC-RAS are similar with MIKE 11 showing a similar profile/trend.

Compared to the steady state RMS error the quasi-steady RMS error for ISIS reduces from 0.098 to 0.097, increases for HEC-RAS from 0.063 to 0.098, and remains the same for MIKE 11 at 0.010, as shown in Table 4.1 and illustrated in Figure 4.1.

Using the transcritical solver in ISIS, the water level is almost identical to the analytical solution, as illustrated in Graph 67. The percentage error is close to zero along the complete length of the channel, as illustrated in Graph 68 and the RMS error is significantly reduced from 0.098 to 0.024.

Using a value of 0.0 for the cross section contraction and expansion coefficients in HEC-RAS for the steady state simulation makes an improvement in the results in the region of supercritical flow and the hydraulic jump, as illustrated in Graphs 57 and 58. The fluctuations in the water level are reduced, although not eliminated, which in turn reduces the RMS error from 0.063 to 0.039.

Using a value of 0.0 for the cross section contraction and expansion coefficients in HEC-RAS for the quasi-steady state simulation makes no change to the results in any way, as illustrated in Graphs 59 and 60. The RMS error remains unchanged at 0.098.

Using the cross section hydraulic radius option in MIKE 11 the RMS error is reduced from 0.010 to 0.095 for the steady and quasi-steady results, as illustrated Figure 4.1 and Table 4.1. The water level and percentage errors being illustrated in Graphs 37 to 40.

4.7 Analysis of Results - Part 6: Transitional Flow

By inspection of Graphs 69 to 72, for the cross-sections at 20.0m, 40.0m, 60.0m and 80.0m respectively, it can be seen that each of the software packages responds in a similar manner for the unsteady transitional flow regime.

No instabilities are observed as the flow changes from supercritical flow to subcritical flow along the lower section of the channel.

The water level for MIKE 11 is predicted to initially rise at a slightly faster rate than that of ISIS and HEC-RAS at each of the four chainages. However, this may be explained by the quasi-steady result for part 5 of the test for both ISIS and HEC-RAS in which a lower water surface profile and a greater percentage error was calculated, when compared to the analytical steady state solution.

5 CONCLUSIONS

Each of the software packages is able to produce a result for all six parts of the test; however, the default calculation options/settings do not necessarily produce the most accurate results when compared to the analytical solution for parts 1 to 5.

ISIS is the most accurate of the three software packages for modelling steady state flows when using the transcritical solver. When the transcritical solver is not used then the results are similar to those calculated by HEC-RAS.

HEC-RAS is the most accurate of the three software packages for modelling quasi-steady flows and by analogy unsteady flows, although, both ISIS and MIKE 11 have only a marginally higher average RMS error.

HEC-RAS consistently produces a RMS error of less than 0.02 for all steady state simulations when the cross section contraction and expansion coefficients are set to zero. Setting these values to zero avoids over estimation of the energy losses and the production of oscillations in the water surface profile in regions of supercritical flow.

With the exception of part 4, the MIKE 11 default steady state and quasi-steady results are the same for both the Resistance Radius and Hydraulic Radius methods respectively.

When using the Hydraulic Radius option in MIKE 11 (Resistance radius is the default) the quasi-steady results are very similar to the ISIS and HEC-RAS results. This is not surprising given that the analytical solution adopts the hydraulic radius approach, which may be considered to be the 'industry standard' approach.

The developers of HEC-RAS have confirmed during this study that the contraction and expansion coefficients are only used in the solution of the energy equation (i.e. steady flow). They are not used in the solution of the momentum equation (unsteady flow). This has been confirmed by the testing undertaken in this study.

Careful consideration needs to be given to the use/value of contraction and expansion coefficients as they can noticeably influence the results.

The use of the default implicit weighting factor of 0.5 in MIKE 11 has led to instabilities in the numerical solutions. However, by increasing this value to 0.55, which is less than the 0.7 and 1.0 default values for ISIS and HEC-RAS, the instabilities are negated. It should be noted that the HEC-RAS manual does recommend that the default value of 1.0 should be reduced once a stable model has been constructed, however, it has been beyond the scope of the test to investigate this.

By using the default implicit weighting factor of 0.5 in MIKE 11 the numerical scheme is fully centred and no numerical dampening is applied to the solution. Hence, artificial dampening is not automatically imposed on the model results, unlike ISIS and HEC-RAS.

With the exception of HEC-RAS, when undertaking a steady state simulation that involves a supercritical upstream boundary (HEC-RAS requires the upstream boundary to be defined in

such instances, although it can be set to critical) each of the software packages consistently underestimates the water level at the upstream boundary.

For parts 2 to 5 of the test each of the software packages consistently underestimates the water level at the upstream boundary when undertaking a quasi-steady calculation.

Differences between the MIKE 11 results and the analytical solution could be explained by the way MIKE 11 suppresses the convective acceleration term for flows approaching supercritical. However, since this study has only considered the default option available within MIKE 11 and not the more advanced option (formulation) it is not possible to qualify or quantify this statement.

6 RECOMMENDATIONS

The novice and even expert modeller often needs the use of a good and well documented reference manual when using the software packages. Furthermore, provision of assistance and the supply of warning messages can often lead to better modelling practice and constancy when building and running a model. From undertaking this test it is believed by the testers that the following improvements to the software packages would benefit the modeller.

Warnings at runtime or in the diagnostics file for HEC-RAS that reminds the user that the expansion and contraction coefficients should be checked when undertaking steady state calculations (i.e. when the default values may not be appropriate).

The use of the default implicit weighting factor of 0.5 in MIKE 11 is a stringent value (a value of 0.55 required in the testing). Although the more expert modeller may commonly increase this value, knowing that it is unlikely, for most non-tidal modelling situations, to have any significant effect on the accuracy of the result, the novice modeller may not readily explore this approach in an attempt to provide a stable solution. Hence, it is recommended, as part of the comments made earlier that improvements could be made that could assist/help the modeller.

The default implicit weighting factor for ISIS, MIKE11 and HEC-RAS are 0.7, 0.5 and 1.0 respectively. An investigation of the appropriateness and impact of these values has been beyond the scope of this study; however, it is recommended that this be investigated.

For clarity it should be noted that the default implicit weighting factor in ISIS and HEC-RAS, which were used in the testing, are 0.7 and 1.0 respectively. The investigation into the appropriateness of these default values has been beyond the scope of this project.

The more advanced formulation in MIKE 11 for suppression of the convective acceleration term for flows approaching supercritical should be investigated as part of further study.

This test should be repeated once the outputs of the Agency's Conveyance Estimator System (CES) R&D study is complete and have been incorporated to the respective software packages. This study is developing a 'Conveyance Generator' that estimates the channel conveyance capacity based on the channel geometry and roughness, which is suitable for in-bank and out-of-bank flow in all UK rivers.

7 REFERENCES

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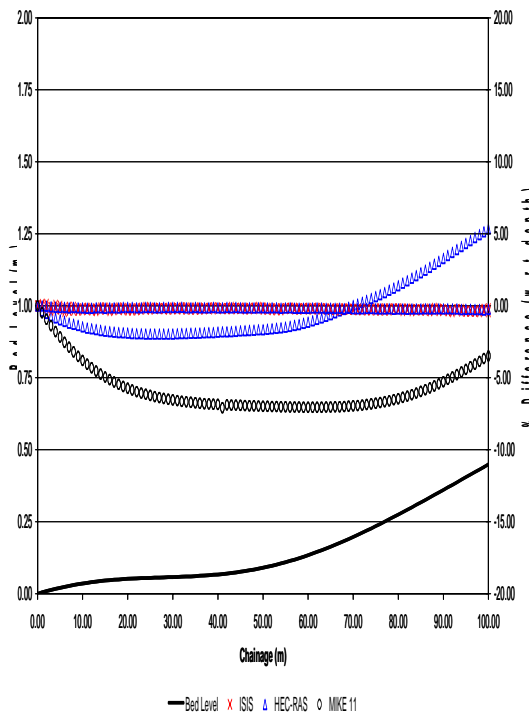
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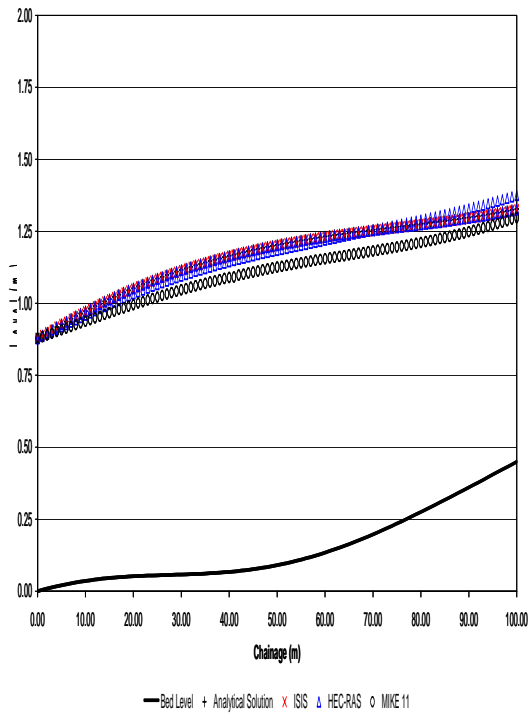
MIKE 11 User Guide, (2000) DHI Software, Denmark

APPENDIX A RESULTS

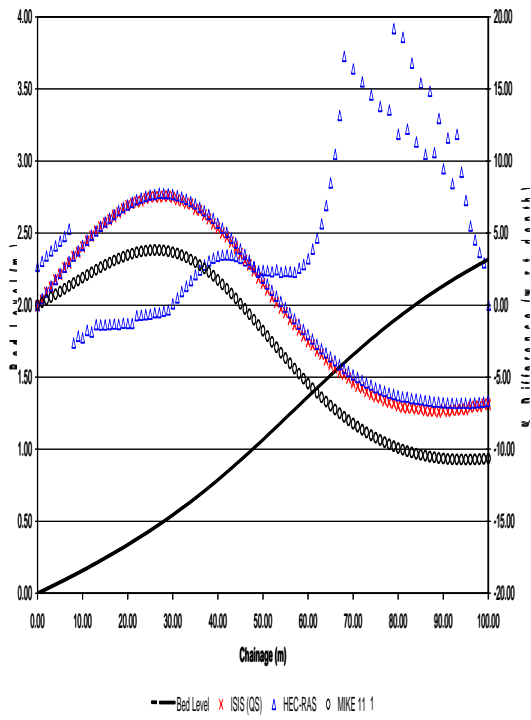
Graph 4- Test A Part 1 (Steady State): Subcritical Flow
% Difference between Analytical and Numerical Results
 Comparison of ISIS, HEC-RAS and MIKE 11



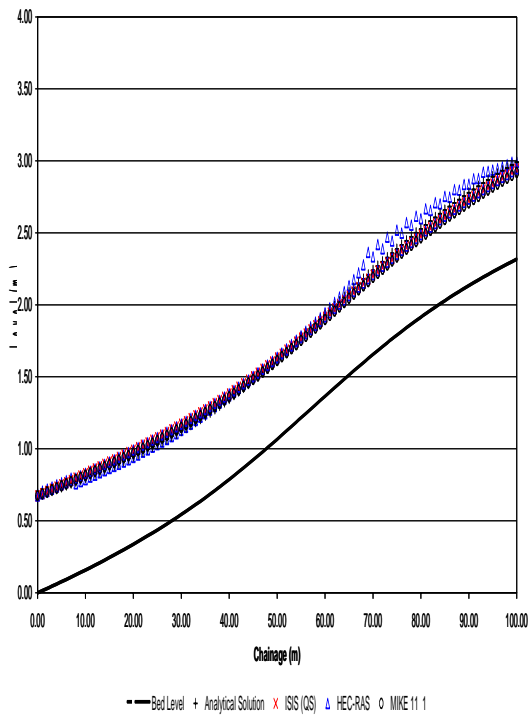
Graph 3- Test A Part 1 (Steady State): Subcritical Flow
Water Levels
 Comparison of ISIS, HEC-RAS and MIKE 11



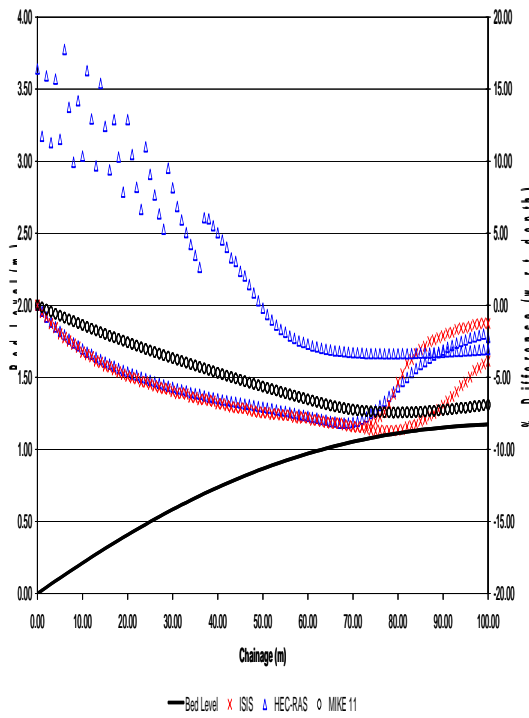
Graph 6 - Test A Part 2 (Steady State): Supercritical Flow
% Difference between Analytical and Numerical Results
 Comparison of ISIS, HEC-RAS and MIKE 11



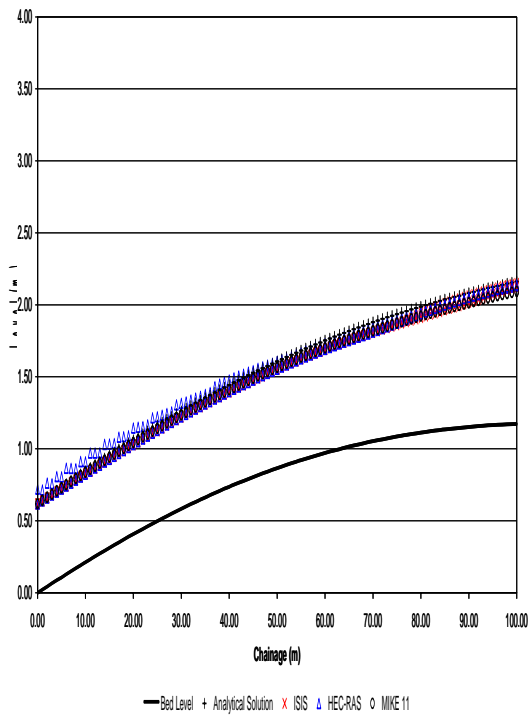
Graph 7 - Test A Part 2 (Steady State): Supercritical Flow
Water Levels
 Comparison of ISIS, HEC-RAS and MIKE 11



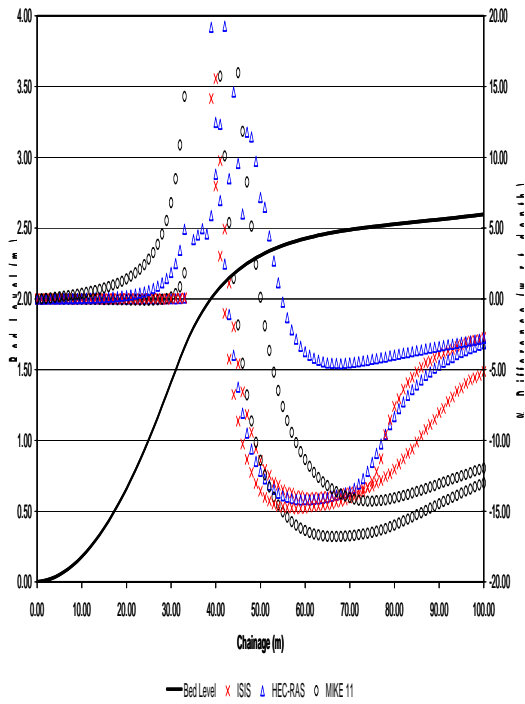
Graph 10 - Test A Part 3 (Steady State): Subcritical to Supercritical Flows
 % Difference between analytical and numerical results
 Comparison of ISIS, HEC-RAS and MIKE 11



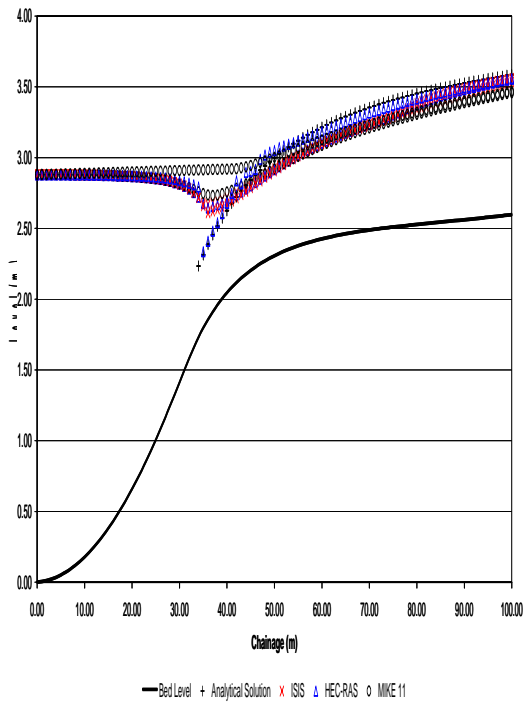
Graph 11 - Test A Part 3 (Steady State): Subcritical to Supercritical Flows
 Water Levels
 Comparison of ISIS, HEC-RAS and MIKE 11



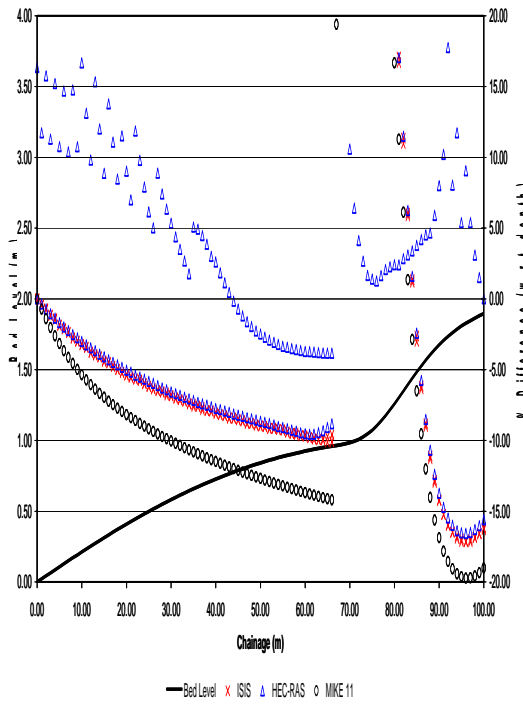
Graph 18 - Test A Part 4 (Steady State): Subcritical to Supercritical to Subcritical Flow
 % Difference between analytical and numerical results
 Comparison of ISIS, HEC-RAS and MIKE 11



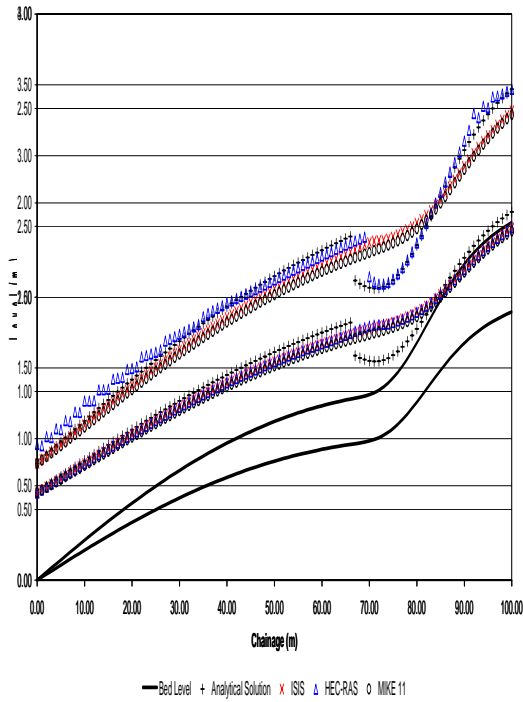
Graph 19 - Test A Part 4 (Steady State): Subcritical to Supercritical to Subcritical Flow
 Water Levels
 Comparison of ISIS, HEC-RAS and MIKE 11



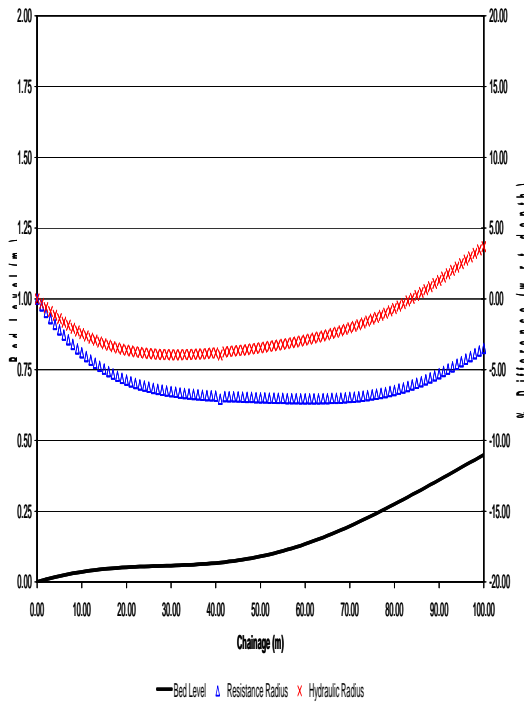
Graph 20 - Test A Part 5 (Steady State): Supercritical to Subcritical to Supercritical Flow
 % Difference between Analytical and Numerical Results
 Comparison of ISIS, HEC-RAS and MIKE 11



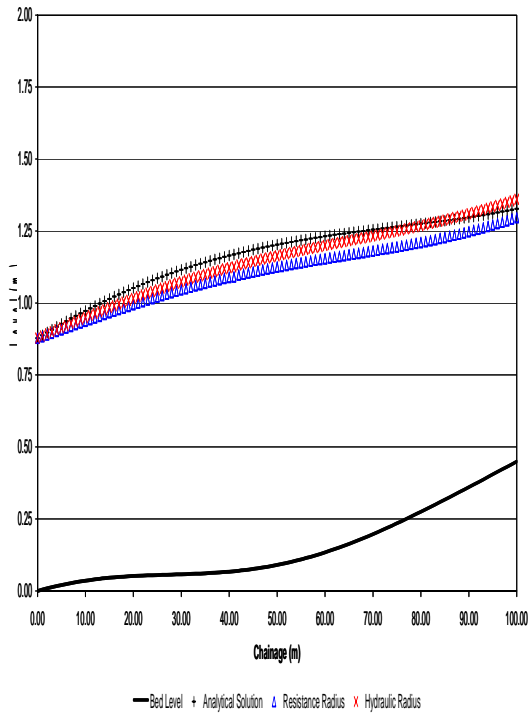
Graph 19 - Test A Part 5 (Steady State): Supercritical to Subcritical to Supercritical Flow
 Water Levels
 Comparison of ISIS, HEC-RAS and MIKE 11



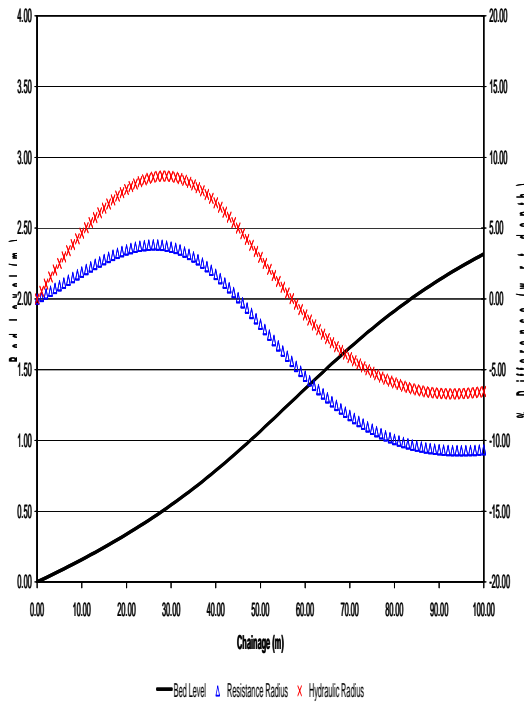
Graph 20 - Test A Part 1 (Steady State): Subcritical Flows
 Difference in Analytical and Numerical Results
 MIKE 11 - Resistance Radius and Hydraulic Radius



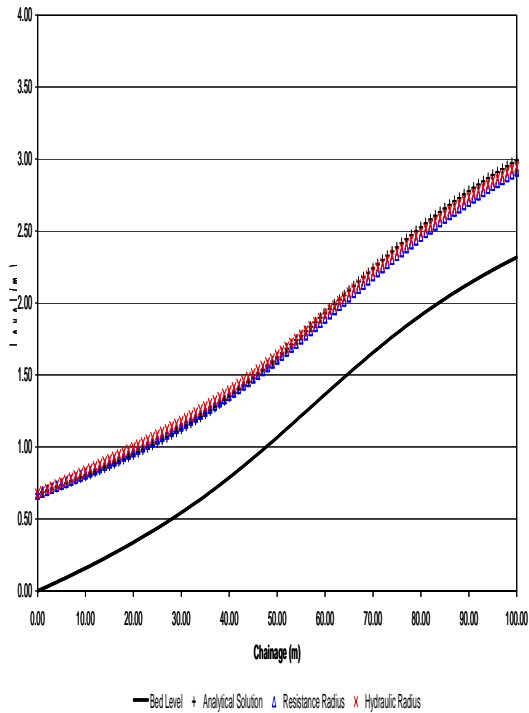
Graph 21 - Test A Part 1 (Steady State): Subcritical Flows
 Water Levels
 MIKE 11 - Resistance Radius and Hydraulic Radius



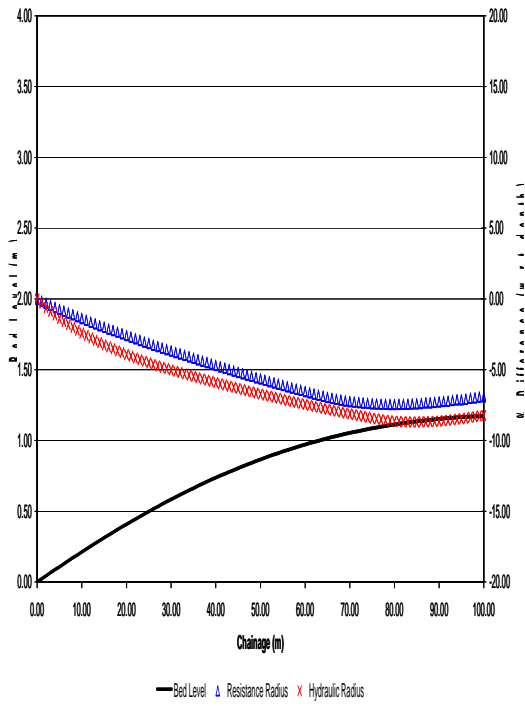
Graph Z6 - Test A Part 2 (Steady State): Supercritical Flows
 Difference in Analytical and Numerical Results
 MIKE 11 - Resistance Radius and Hydraulic Radius



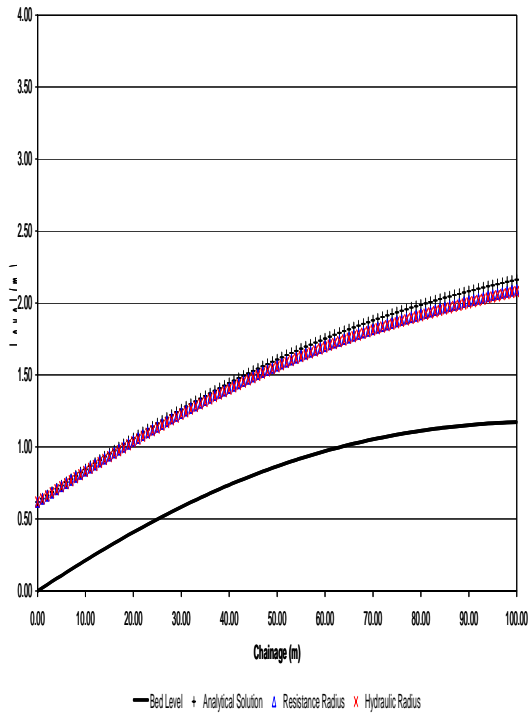
Graph Z6 - Test A Part 2 (Steady State): Supercritical Flows
 Water Levels
 MIKE 11 - Resistance Radius and Hydraulic Radius



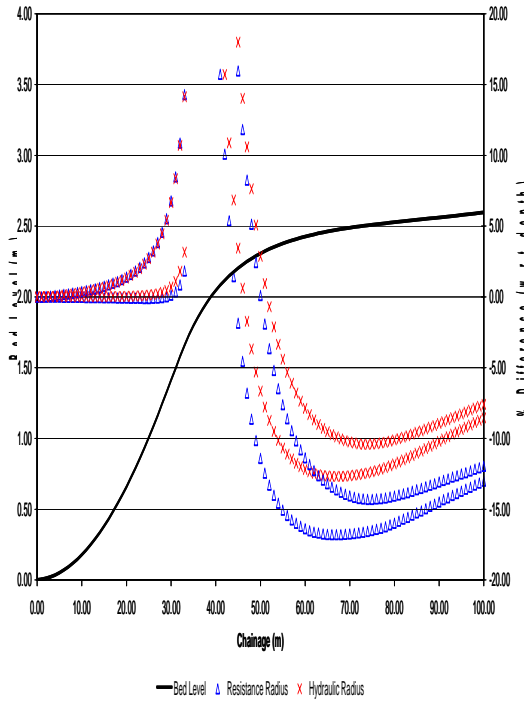
Graph 30 - Test A Part 3 (Steady-State): Supercritical to Subcritical Flow
 % Difference in Analytical and Numerical Results
 MIKE 11 - Resistance Radius and Hydraulic Radius



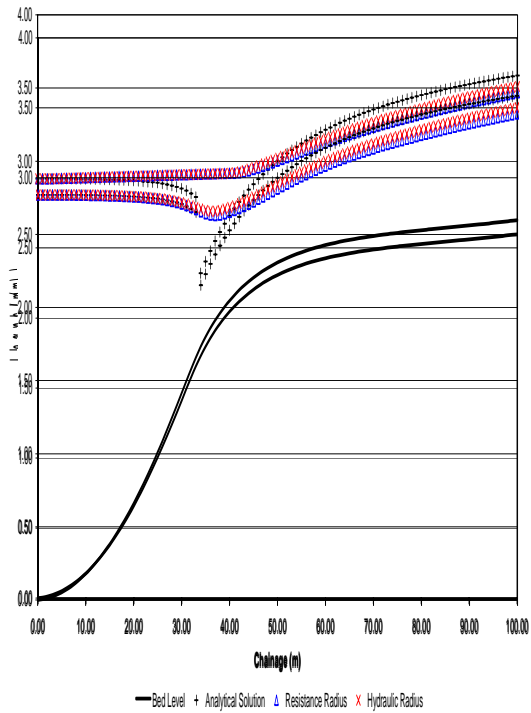
Graph 29 - Test A Part 3 (Steady-State): Supercritical to Subcritical Flow
 Water Levels
 MIKE 11 - Resistance Radius and Hydraulic Radius



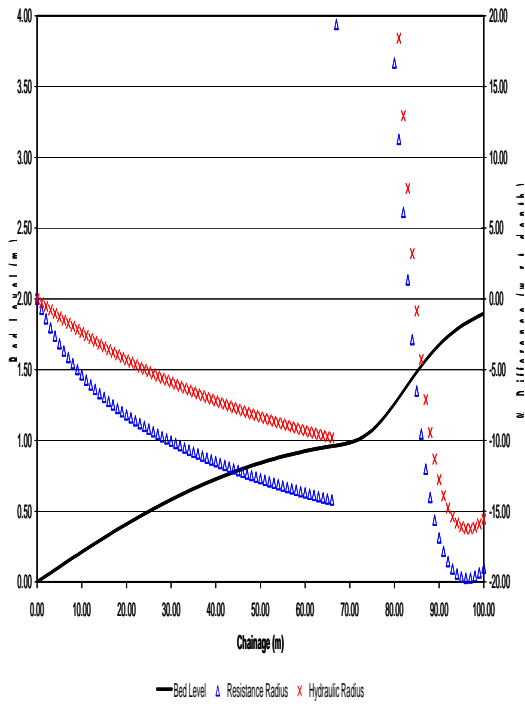
Graph 38 - Test A Part 4 (Steady State): Subcritical to Supercritical to Subcritical Flow
 % Difference in Analytical and Numerical Results
 MIKE 11 - Resistance Radius and Hydraulic Radius



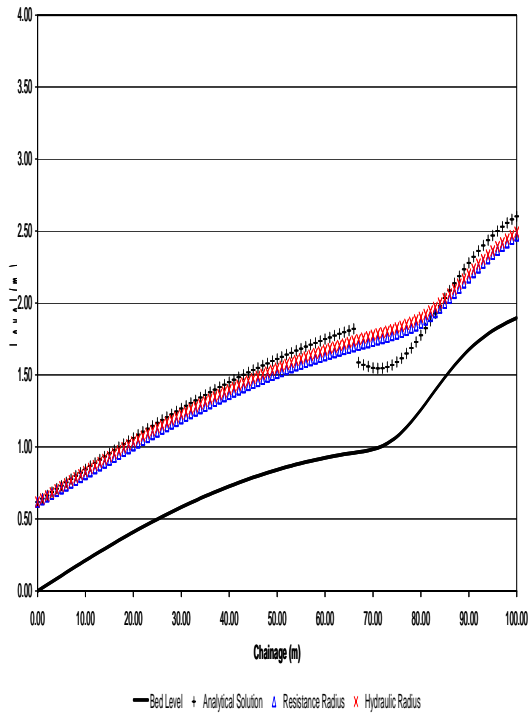
Graph 33 - Test A Part 4 (Steady State): Subcritical to Supercritical to Subcritical Flow
 Water Levels
 MIKE 11 - Resistance Radius and Hydraulic Radius



Graph 40 - Test A Part 5 (Steady State): Supercritical to Subcritical to Supercritical Flow
 % Difference in Analytical and Numerical Results
 MIKE 11 - Resistance Radius and Hydraulic Radius



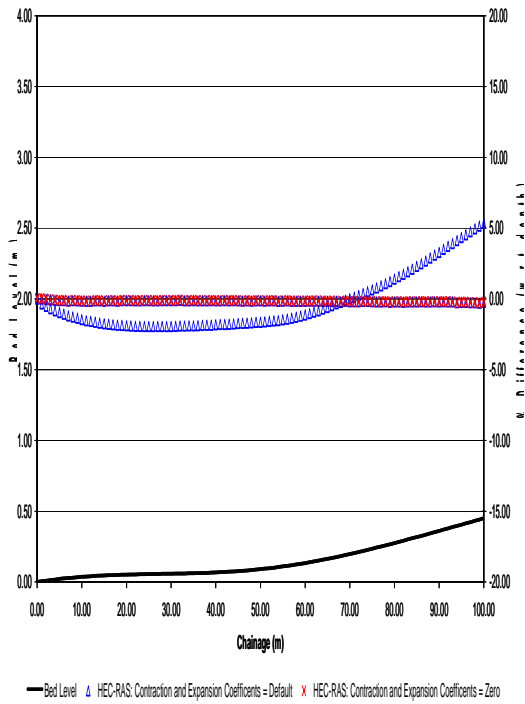
Graph 39 - Test A Part 5 (Steady State): Supercritical to Subcritical to Supercritical Flow
 Water Levels
 MIKE 11 - Resistance Radius and Hydraulic Radius



Graph 42 - Test A Part 1 (Steady State): Subcritical Flows

Difference in Analytical and Numerical Results

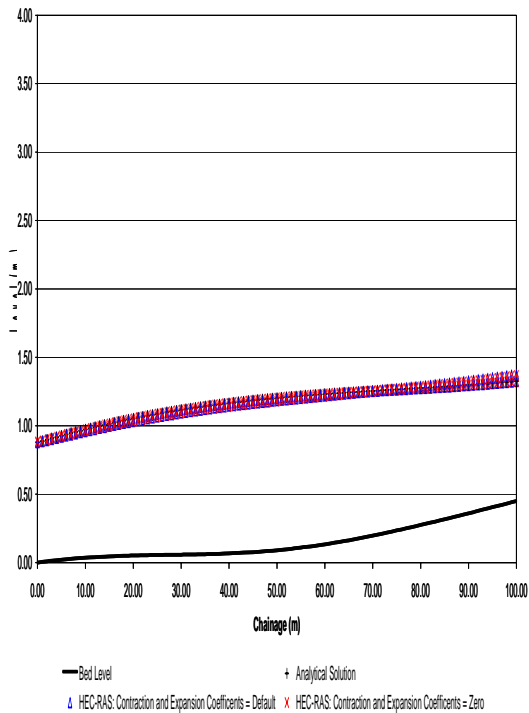
HEC-RAS with contraction and expansion coefficients set at their default values and at zero



Graph 43 - Test A Part 1 (Steady State): Subcritical Flows

Water Levels

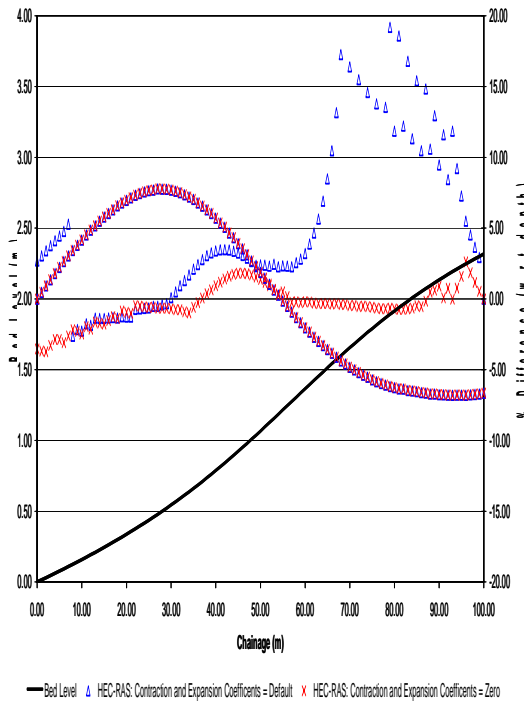
HEC-RAS with contraction and expansion coefficients set at their default values and at zero



Graph 46 - Test A Part 2 (Steady State): Supercritical Flows

Difference in Analytical and Numerical Results

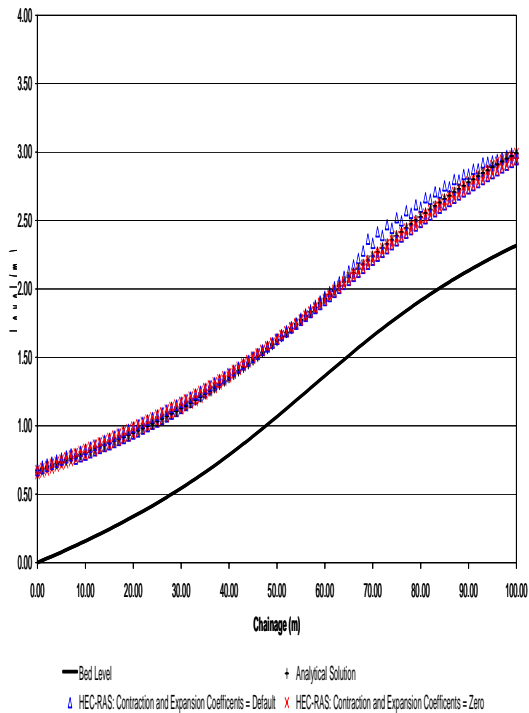
HEC-RAS with contraction and expansion coefficients set at their default values and at zero



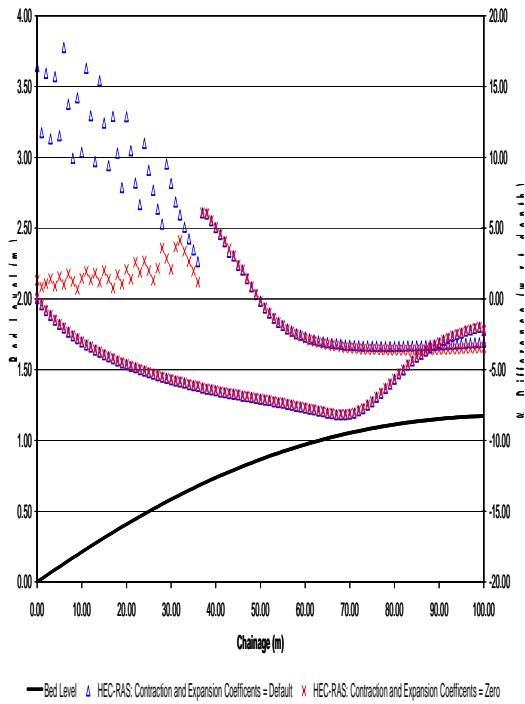
Graph 45 - Test A Part 2 (Steady State): Supercritical Flows

Water Levels

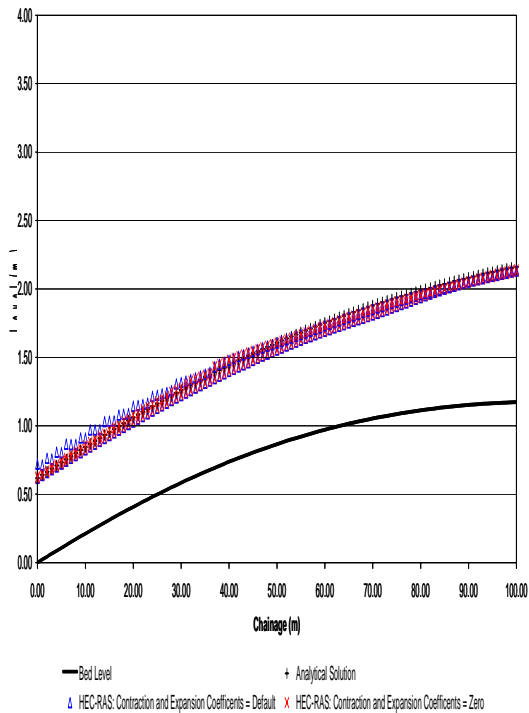
HEC-RAS with contraction and expansion coefficients set at their default values and at zero



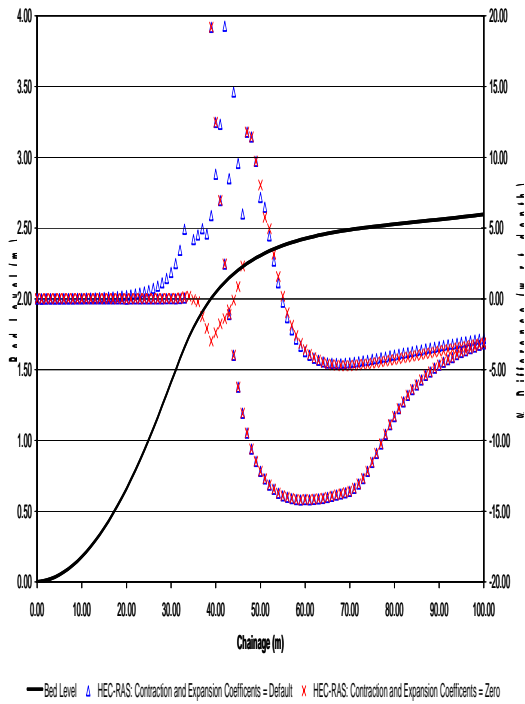
Graph 5D - Test A Part 3 (Steady State): Supercritical to Subcritical Flow
% Difference in Analytical and Numerical Results
 HEC-RAS with contraction and expansion coefficients set at their default values and at zero



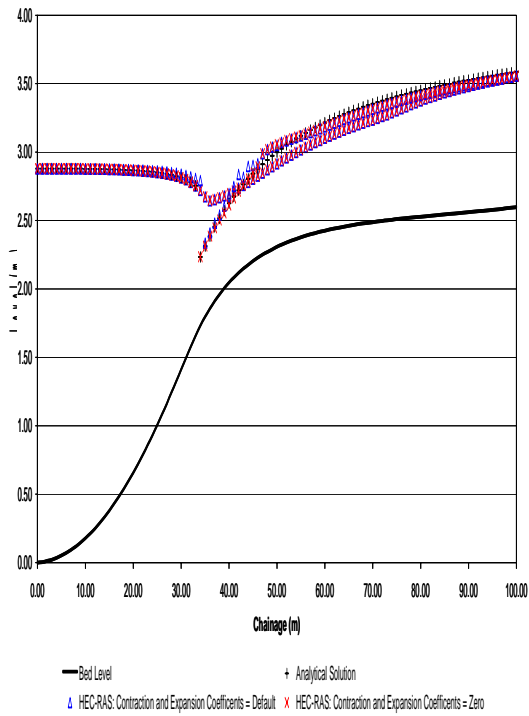
Graph 5E - Test A Part 3 (Steady State): Supercritical to Subcritical Flow
Water Levels
 HEC-RAS with contraction and expansion coefficients set at their default values and at zero



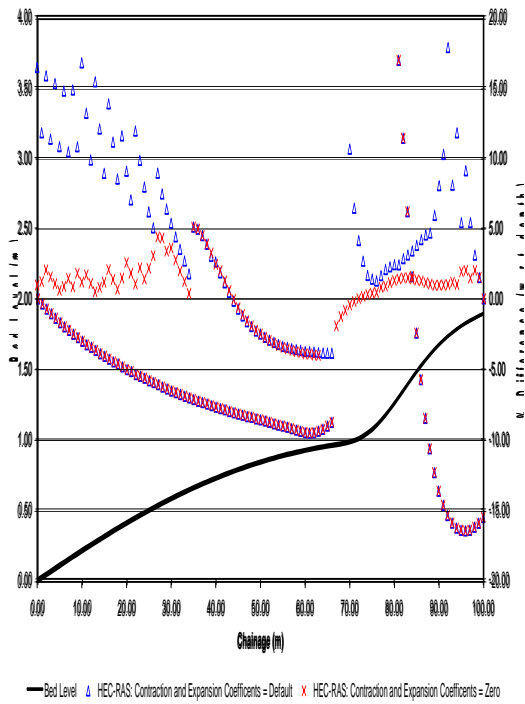
Graph 58 - Test A Part 4 (Steady State): Subcritical to Supercritical to Subcritical Flow
% Difference in Analytical and Numerical Results
 HEC-RAS with contraction and expansion coefficients set at their default values and at zero



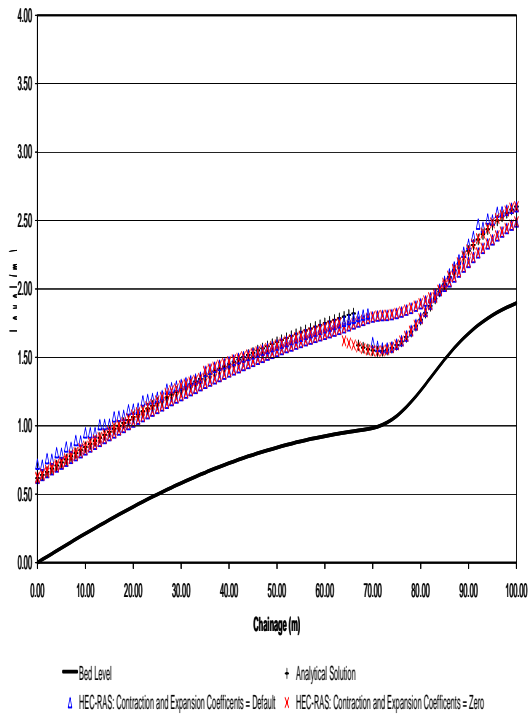
Graph 59 - Test A Part 4 (Steady State): Subcritical to Supercritical to Subcritical Flow
Water Levels
 HEC-RAS with contraction and expansion coefficients set at their default values and at zero



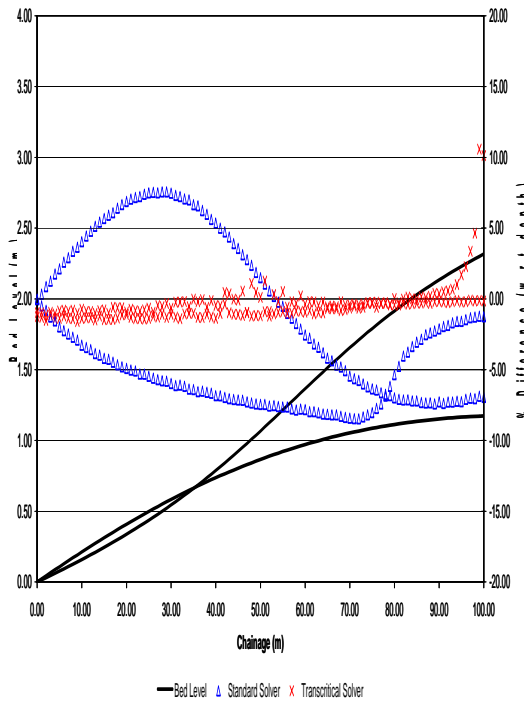
Graph 68 - Test A Part 5 (Quasi Steady): Supercritical to Subcritical to Supercritical Flow
% Difference in Analytical and Numerical Results
 HEC-RAS with contraction and expansion coefficients set at their default values and at zero



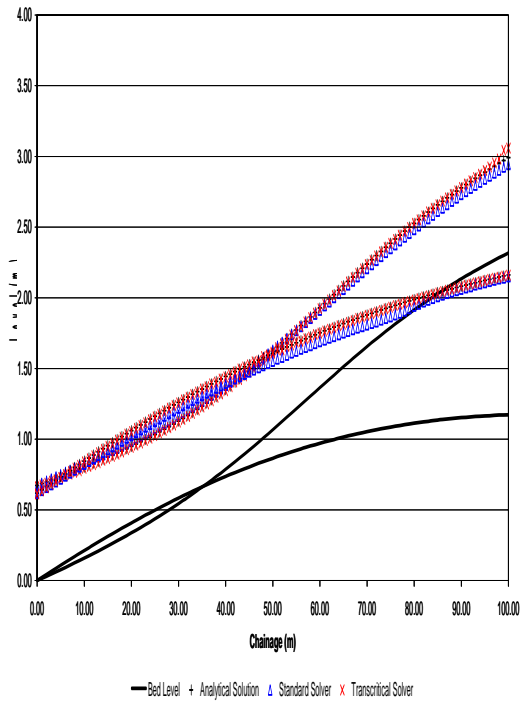
Graph 69 - Test A Part 5 (Steady State): Supercritical to Subcritical to Supercritical Flow
Water Levels
 HEC-RAS with contraction and expansion coefficients set at their default values and at zero



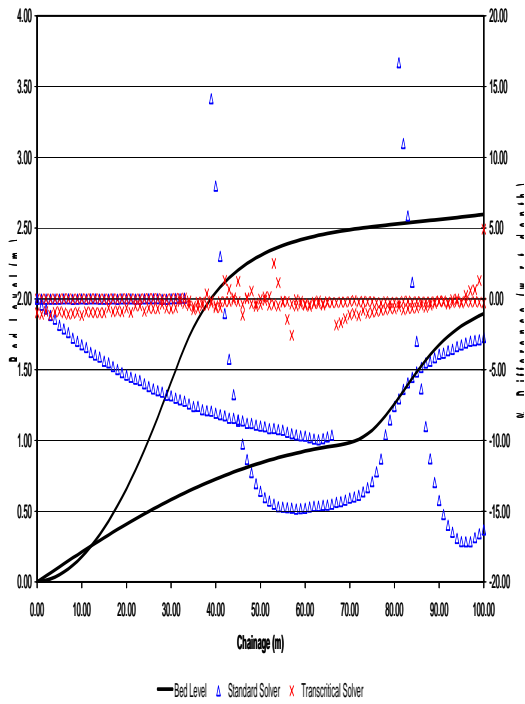
Graph 6 Step 41 A Part A Part 2 (State) Step 41 in Bed Elevation Flow
 % Difference in Analytical and Numerical Results
 ISIS - Standard and Transcendental Solvers



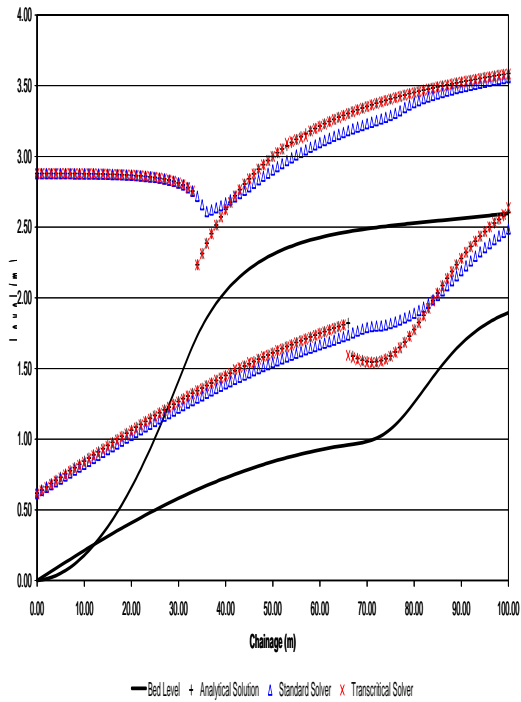
Graph 6 Step 41 A Part A Part 2 (State) Step 41 in Bed Elevation Flow
 Water Levels
 ISIS - Standard and Transcendental Solvers



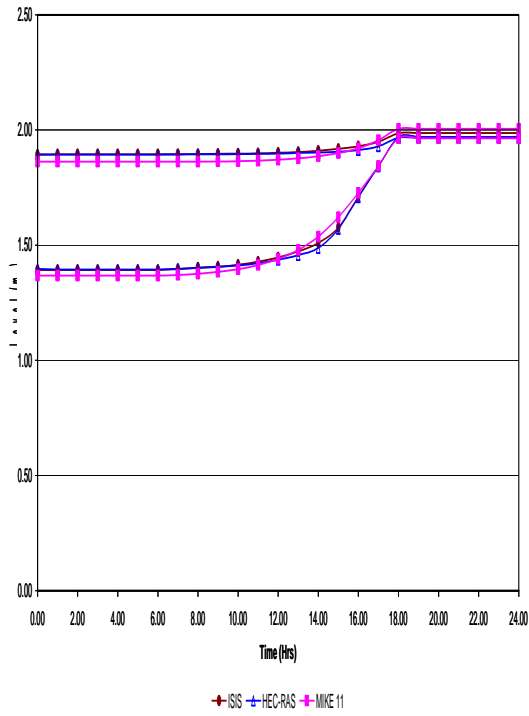
Graph 66-Test4Part 6 (Steady State) Subcritical to Supercritical to Subcritical Flow
 % Difference in Analytical and Numerical Results
 ISIS - Standard and Transcritical Solvers



Graph 65-Test4Part 6 (Steady State) Subcritical to Supercritical to Subcritical Flow
 Water Levels
 ISIS - Standard and Transcritical Solvers



Graph 70 - Test A Part 6 (Unsteady): Transitional Flow
 Water Levels at 80.0m
 Comparison of ISIS, HEC-RAS and MIKE 11



Graph 89 - Test A Part 6 (Unsteady): Transitional Flow
 Water Levels at 80.0m
 Comparison of ISIS, HEC-RAS and MIKE 11

