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Alex Burgerman and Nishaal Gooroochurn  
HMRC-KAI / Computable General Equilibrium Modelling  
100 Parliament Street / London /SW1A 2BQ

October 11, 2013

Dear Alex and Nishaal:

Please find attached our *CGE Peer Review of Tax Model*.

If you have further questions, please do not hesitate to contact us.

Yours sincerely,

Christoph Böhringer and Thomas F. Rutherford  
*Loch Alpine Economics, Inc.*

# ***CGE Peer Review of Tax Model***

## **Summary**

Over the last two decades, computable general equilibrium (CGE) models have become a standard numerical framework for quantifying the economy-wide impacts of policy reforms. CGE models build upon general equilibrium theory that combines assumptions regarding the optimizing behaviour of economic agents with the analysis of equilibrium conditions: producers combine primary factors and intermediate inputs at least cost subject to technological constraints; given preferences consumers maximize their well-being subject to budget constraints. CGE analysis provides counterfactual ex-ante comparisons, assessing the outcomes with a reform in place with what would have happened had it not been undertaken. The main virtue of the CGE approach is its comprehensive representation of price-dependent market interactions based on rigorous microeconomic theory. The simultaneous explanation of the origin and spending of the agents' incomes makes it possible to address both economy-wide efficiency as well as distributional impacts of policy interventions.

HMRC uses a multi-sector dynamic Ramsey model of intertemporal saving and investment (thereafter referred to as HMRC model) to assess the impacts of fiscal policy reforms for the UK economy. Our review of the HMRC model package finds that its basic implementation meets the requirements of state-of-the-art CGE-based tax policy analysis. It is suitable for quantifying the economic adjustment paths triggered by alternative tax policy proposals and thereby help UK policy makers putting their decisions about fiscal reforms on a more informed basis. However, we suggest a couple of short-term modifications and mid-term extensions to enhance the usefulness of the modelling framework for applied UK tax policy analysis.

Our peer review is organized as follows. In section 1 we sketch important effects of tax policy reforms that must be captured by quantitative methods for impact assessment and then discuss in more detail the key requirements of CGE-based tax policy analysis. In section 2 we provide a critical appraisal of how the HMRC model meets the requirements for state-of-the-art quantification of economic impacts triggered by tax policy changes. In section 3 we recommend short-term modifications to avoid potential inconsistencies and improve on the computational performance and robustness of the HMRC model package. In section 4 we list mid-term model extensions that could substantially enhance and broaden the scope of applicability of the HMRC model.

## 2. Key requirements for economic impact assessment of tax policy reforms

From a public finance perspective, taxes primarily serve the purpose of raising stable revenues in order to cover public spending for transfers and the provision of public goods. Adopting the neoclassical efficiency paradigm of competitive markets, the tax system should preserve the pre-tax efficient allocation of scarce resources as far as possible. In this vein, a central challenge of fiscal policy is the reform of the existing tax system in order to reduce distortionary effects on overall economic performance. The distortionary impact of a tax can be characterized by its so-called marginal cost of public funds (MCPF) which indicates the economy-wide cost associated with an additional monetary unit of revenue raised by the specific tax. For instance, an MCPF of 150 % for a tax means that for any additional pound of public revenue there is a money-metric economy-wide loss of one and a half pounds – the marginal excess burden here would amount to 50 pence. An optimal tax system under pure efficiency considerations boils down to equalizing the MCPF of different taxes at the lowest level possible. Textbook economics indicates that the excess burden of a (distortionary) tax increases with the square of its rate and linearly depends on the supply or demand elasticity of the taxed commodity. A rough guideline for efficient taxation thus is to broaden the tax base (in order to reduce tax rates for a given tax revenue) and pick commodities whose supply or demand is rather inelastic.

Beyond efficiency considerations, tax policy in real practice has to account for inherently normative distributional goals such as the “fair” incidence of the tax burden across households with different incomes. In the absence of lump-sum transfers, equity norms which reflect some degree of inequality aversion typically lead to a tax system which is suboptimal under pure efficiency considerations (i.e. a perspective which is agnostic on the distribution of costs and benefits). A prime example for the trade-offs between efficiency and equity is value-added taxation. A uniform consumption tax rate which is relatively efficient from a revenue-raising point of view may be rejected on equity grounds because of its regressive effects.

In order to investigate the implications of tax policy reforms it is useful to distinguish between tax-interaction effects and revenue-recycling effects. A new tax does not only affect the resource use in markets in which it is applied but also in other markets. Interaction with existing taxes induces additional gross costs for any given use of the additional revenue if the initial tax system is optimal in terms of raising a given amount of revenues with the lowest cost to private income. To raise the same amount of revenue by the new tax gets more costly. One example of a tax-interaction effect is the tax base erosion when revenue-neutral tax swaps do not only adversely change their own tax base but also the tax bases of other taxes. If the existing tax system is inefficient, the introduction of additional taxes can either exacerbate or reduce pre-existing tax distortions. The existence of initial distortionary taxes also provides scope for efficiency improvements though tax recycling of additional revenues raised by a new tax. In a second-best setting with existing distortionary taxes a rebate of the tax revenue through cuts in initial distortionary taxes will reduce the gross cost of a new tax relative to a lump-sum

replacement. It can be shown that revenue recycling by reducing existing distortionary taxes is equivalent to substituting at the margin a lump-sum tax for a distortionary tax. The revenue-recycling effect in terms of efficiency gains is clearly positive.

CGE models are obviously a prime candidate for the quantitative impact assessment of tax policy reforms. They provide a comprehensive analytical framework to capture direct and indirect tax interaction and revenue recycling effects thereby accounting for price-dependent interactions of all major commodity and factor markets as well as the origination and spending of income by economic agents. CGE models do not only deliver positive information on policy-induced changes in key economic indicators at the macroeconomic level (e.g. GDP, investment, consumption, tax revenues), at the sector level (e.g. production, export, import) and at the household level (e.g. income and expenditure), but also allow for normative rankings of alternative policy reforms compared to the status-quo, as they are built upon rigorous microeconomic foundations.

However, to make the transition from stylized to policy-relevant analysis CGE models need not only build on representative empirical data but also incorporate key drivers of economic adjustment to tax policy changes including:

#### Heterogeneity in production and consumption

It is important to account for sector-specific differences in factor intensities, degrees of factor substitutability and the price elasticities of output demand to trace back the structural change in production which is induced by tax policy shifts. Likewise, disaggregation should allow for the identification of tax-induced winners and losers at the sector level since this is a basic requirement for assessing social acceptability and optional compensation mechanisms. Changes of the tax system typically involve a shift of the tax burden borne by factor (resource) owners. Depending on initial endowments and consumption patterns the imposition of new taxes as well as the recycling of tax revenues will cause regressive or progressive effects for consumer groups. To identify conflicts of interests among consumer groups in the decision how to raise taxes and how to recycle tax revenues a distinction between household groups which differ in preferences and income source is necessary. Ultimately, the incorporation of heterogeneous households poses the challenge of weighing the individual costs and benefits across different consumer groups by means of a social welfare function to derive normative conclusion on the desirability of policy reforms.

#### Representation of substitution possibilities in production and consumption

Total analysis of a national economy requires methods for the aggregate description of the production and consumption side. Typically smooth functions are employed which capture (transformation) substitution possibilities through elasticities of substitution (transformation). The elasticities determine the ease of adjustment in input demand to relative price changes triggered by tax policy changes. Empirical estimates for these parameters are crucial for the credible evaluation of the quantitative

effects of alternative tax reform scenarios. There is a trade-off between the flexibility of functional forms, their economic interpretation ("economic well-behaviour") and data availability. Wide-spread functional forms such as nested constant-elasticity-of-substitution (CES) cost and expenditure functions provide sufficient flexibility while being consistent to rationality assumptions on the optimizing behaviour of agents.

### Dynamics

A dynamic dimension is necessary to account for tax-induced changes in saving and investment decisions of firms and households over time. Consumers have a choice between current consumption and leisure versus future consumption which can be purchased via savings. In production, capital (stock) accumulation and reallocation takes time. In their investment decisions, firms consider adjustment costs related to the introduction of new capital. Adjustment costs capture the incomplete mobility of capital across industries as well as the installation cost of capital (i.e., the costs of adjusting capital toward its optimal level). Usually adjustment costs are conceived as internal to the firm and measured in terms of foregone output. The integration of dynamic features in the behaviour of producers is a prerequisite to capture the effects of tax reforms on the allocation of savings, investment decisions and on adjustment costs. Consistency with the basic equilibrium concept (agents have preferences over alternative allocations and they behave in a rational manner) demands perfect foresight – i.e., agents are perfectly knowledgeable about future prices. An intertemporal model setting accommodates dynamic scoring where the effects of fiscal policy reforms are based on rational expectations of economic agents over future prices. Compared to a static framework, the dynamic model specification captures the transition path to the final equilibrium outcome of a tax reform. For policy acceptability, the duration and the strength of transitional impacts can be even more crucial than the long-term equilibrium effects.

### Incorporation of initial tax distortions and transfers

The impacts of tax policy shifts depend fundamentally on the nature of the pre-existing taxes which can be characterized in terms of their marginal cost of public funds (MCPF). Large differences in the MCPF of initial taxes indicate scope for efficiency gains through tax cuts in the most distortionary taxes. The MCPF of initial taxes is central for the design of tax reforms that aim at reducing the economy-wide cost of raising public funds. The distortionary effects of existing taxes are fundamentally determined by elasticities that govern the price-responsiveness of demand and supply reactions. For example, the larger the intertemporal elasticity of substitution in consumption (future consumption versus current consumption) the higher is the distortion of a given capital tax. Tax revenues are used to finance public good provision as well as monetary transfers between the government and private economic agents. Comprehensive tax policy analysis thus requires an appropriate representation of public good provision and transfer payments.

## 4. Critical appraisal of the HMRC CGE model

The basic design of the HMRC model for the UK economy meets at large the key requirements for state-of-the-art applied tax policy analysis. The disaggregation of production and consumption in multiple production sectors and multiple household types allows for the quantification of policy-induced structural change and differential income effects. A dynamic setting provides insights into economic adjustment path induced by economic policy measures. The model is calibrated to empirical data on base-year economic flows including pre-existing taxes and transfers.

The core model logic adopts an intertemporal Ramsey framework of exogenous growth which is appropriate to study the adjustment path and the long-term economic impacts of tax reforms. Investment is driven by consistent expectations of future returns with quadratic adjustment costs. In equilibrium, investments are placed in sectors where they will receive the highest return. Total labour endowment increases with labour force efficiency at an exogenous growth rate. Capital stocks evolve through geometric depreciation and new investment.

The standard model formulation is based on constant-returns-to-scale nested CES functions reflecting competitive market structures. Final demand is characterized by budget-constrained intertemporal optimization of households. Households choose to allocate lifetime income across consumption in different time periods. In each period a household faces the choice between current consumption (non-leisure consumption goods and leisure) and future consumption which can be purchased via savings. The pure rate of time preference determines the intertemporal allocation of consumption. A separable intertemporal utility function is employed where the intra-period utility from consumption is based on a nested CES function over leisure and non-leisure consumption commodities.

The government distributes transfers and provides a public good (including public investment) which is produced with commodities purchased at market prices. Government expenditures are financed with tax revenues. The model incorporates the key features of the UK tax system including income taxes, value-added taxes, production taxes and subsidies. The public budget is balanced on a period-by-period basis according to various closure rules.

In international trade, domestic and foreign goods are distinguished by origin. This accommodates both imports and exports of the same commodity. Demands for imports stem from cost-minimizing producer behaviour and utility maximization of households. On the export side, products destined for domestic and international markets are treated as imperfect substitutes subject to a constant elasticity of transformation. The UK is treated as small relative to the world market. The small country assumption implies that in the model changes in the level of UK exports and imports have no effect on terms of trade - international prices are exogenous in foreign currency, i.e., export demand and import supply functions are horizontal. A variable exchange rate reconciles the present value of domestic import and foreign export demands for each period.

Total labour endowment increases with labour force efficiency at an exogenous growth rate which provides the steady-state growth path to which the model is calibrated. Quantitative analysis of tax policy reforms are measured with respect to the steady-state growth path which reflects the business-as-usual where no policy changes apply.

To perform numerical simulations, the concrete forms of the production functions (characterizing the technological options in production) and the utility functions (characterizing the consumption preferences of households) must be specified. The parameterization procedure most commonly used in CGE analysis and likewise adopted for the HMRC model is known as calibration. Calibration of the free parameters of functional forms requires a consistent data set for a specific base-year in prices and quantities together with exogenous elasticity's that are usually taken from literature surveys. The calibration is a deterministic procedure and does not allow for statistical test of the model specification.

Technically, the model and data routines are implemented in GAMS (Generalized Algebraic Modelling System) using MPSGE (Mathematical Programming System for General Equilibrium). GAMS is a higher-level model language for the development of large-scale mathematical programs and the processing of extensive datasets. MPSGE which works as a subsystem in GAMS is a modelling language specially designed for solving Arrow-Debreu economic equilibrium models. It provides a transparent and relatively painless way to write down and analyse complicated systems of nonlinear inequalities. The language is based on nested constant-elasticity-of-substitution (CES) utility functions and production functions. The data requirements for a model include share and elasticity parameters for all the consumers and production sectors included in the model. Typically, these are calibrated from a consistent benchmark equilibrium dataset. When used by CGE experts, MPSGE reduces the setup cost of producing an operational model and the cost of testing alternative specifications. For data management, GAMS provides a convenient interface to EXCEL. Original data can be directly imported into GAMS where subsequent data refinements and modifications can be transparently controlled – model output on the other hand can be readily exported to EXCEL hereby making use of the Pivot-Table/-Chart functionality for multi-dimensional presentation of results.

It should be noted that providing robust analysis requires the modeller to complete a number of steps: checking data and model calibration, coding the logic for policy scenarios of interest, and last but not least carrying out comprehensive sensitivity analysis to check robustness of simulation results with respect to changes in model assumptions and data. These processes can be time consuming, demanding for weeks up to months before completion of analysis.

## **5. Short-term modifications**

The peer review of the HMRC model package suggests a few modifications in the model logic to avoid potential inconsistencies and improve on the computational performance and robustness of the HMRC model package.

#### Specification of transfers

In the current model formulation transfers between the government and the private agents are expressed through a specific transfer good for which supply and demand arises solely through endowment entries. This can lead to unreliable model results. The point is that if a commodity does not have an inherent value determined through markets, the implicit value of transfers of that commodity is undefined. To fix the problem we suggest denominating transfers in terms of a marketed good such as the composite consumption good or foreign exchange.

#### Terminal constraints

Dynamic general equilibrium models exhibit a turnpike property. One can exploit this when an infinite horizon equilibrium must be approximated with a finite horizon model. To assure invariance of model results with respect to the time horizon, a set of appropriate terminal conditions must be specified. We recommend the inclusion of post-terminal capital stocks for each sector as endogenous variables. Using state variable targeting for these variables, the growth of investments in the terminal period can be related to the stable long-term growth rate of the economy. These constraints impose balanced growth in the terminal period but do not require that the model achieves steady-state growth. Furthermore, the choice of the time horizon must reflect the stringency of the policy shock under investigation.

#### Baseline calibration

The model is calibrated to steady-state growth path which seems appropriate for the analysis of fiscal tax reforms given the exogenous growth prescriptions of the dynamic Ramsey framework. A central challenge involved in calibrating a dynamic model centres on the reconciliation of base-year capital earnings, investment, the steady-state interest rate, the exogenous growth rate and the capital depreciation rate. Observed base-year values for capital earnings and investment typically are not consistent with arbitrary values of growth, interest and depreciation rates. Something usually has to be adjusted to match up the dataset with the baseline growth rate. In the current HMRC calibration sector-specific risk premiums are used for this adjustment leading to a wide range of positive and negative mark-ups on the risk-free reference rate of return. We recommend revising the adjustment procedure towards changes in implied base-year capital earnings.

#### Labour-leisure choice

Labour supply in the standard model can be either treated as fixed or endogenous. In the latter case, labour supply is elastic through demand for leisure which enters intra-period utility of households on a

nested CES function over leisure and non-leisure consumption commodities. We recommend calibration of the CES elasticity in line with empirical estimates on the elasticity of labour supply with respect to the real wage.

#### Documentation and streamlining of model/data routines

The current model package is poorly documented – both with respect to direct comments and explanations in the programming code as well as with respect to technical documentation for non-programmers. The review suggests that the model and the data underlying its parameterization should be thoroughly documented - via in-line documentation throughout the programming code and via technical reports. Furthermore, there is scope for a more proficient organization of data input routines, model code and results reporting. This is an important pre-requisite for both the internal workforce at HMRC as well as cooperating external consultants to make efficient use of the model package and develop it further according to customers' needs.

## **5. Mid-term extensions**

There are a couple of mid-term extensions that could substantially enhance the scope of applicability of the HMRC model package. The extensions are listed below:

- Refinement of debt-/equity-financing and treatment of risk
- Calibration of a linear-expenditure system
- Extensions of the core model to feature imperfect competition for selected sectors
- Refined representation of initial energy taxes and subsidies
- Reduced form implementation of involuntary unemployment (e.g. wage curve)
- Alternative representation of imperfectly competitive market structures
- Overlapping generations (intergenerational burden sharing)