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# Advice on Statistical Confidence of Appraisal Non-Work Values of Time

## Final Report

Phill Wheat, Mark Wardman and John Bates (John Bates Services)

Institute for Transport Studies (ITS)
University of Leeds
Tel (direct): + 44 (0) 113 34 35344
Tel (enquires): +44 (0)113 34 35325
Fax: + 44 (0) 113 34 35334

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ITS

# 1. INTRODUCTION

The Institute for Transport Studies with John Bates Services has been commissioned by the Department for Transport to provide advice on the statistical confidence of appraisal non-work values of travel time (VoTT). The DfT wishes to understand the uncertainty arising from its current VoTT estimates in order to inform decision makers as to the likely benefit from re-sampling and re-estimating such values.

The current VoTT estimates are computed with reference to the output of two statistical models, in the following way :

1) In 1994, Stated Preference data for VoTT estimation in the highway context was collected by Accent Marketing and Hague Consulting Group [AHCG]). Although they produced an analysis and recommendations (AHCG, 1999), these were not implemented. Subsequent re-analysis by ITS/Bates (Mackie et al, 2003[1]) concluded that VoTT was a function of income and journey cost (interpreted as trip distance). Separate analyses were conducted for Commuting and “Other” leisure travel.
2) The resulting model formula for VoTT was then used to predict the value of VoTTs for various income and trip distance combinations. An overall distance-weighted average was obtained by weighting the combinations according to the distribution (for all mechanised modes) in the NTS 1995-2000 data.
3) It was deemed that the income elasticity from the Stated Preference model was not suitable for forecasting going forward in time since the SP was based on cross sectional data
4) A further model, based on meta data, was deemed to produce a more appropriate income elasticity
5) The base VoTT was then up rated by applying the income elasticity from the meta model

The approach taken in this study is to make separate allowance for the uncertainty implicit in both models, and to combine them appropriately. The output of the work is a spreadsheet which computes confidence intervals for VoTT. This report represents the supporting document for the spreadsheet.

[1] Mackie, P, Wardman, M, Fowkes, A, Whelan, G, Nellthorp, J and Bates, J (2003). *Values of Travel Time Savings*. Report for the Department for Transport, Great Britain.

Following this introduction, Section 2 outlines the methodology used to compute the confidence intervals. Section 3 provides an overview of the code for the [R] matrix manipulation package which has been used to provide standard errors for the base VoTT. Section 4 provides an overview of the spreadsheet which draws on this standard error estimate and discusses associated functionality. Section 4 concludes with reference to how the spreadsheet can be amended to compute uncertainty for different estimates should DfT commission a new VoTT study.

## 2. METHODOLOGY

We can identify four key sources of statistical uncertainty influencing the up rated VoTT estimates

1) Parameter uncertainty arising from estimation of the logit model – this yields uncertainty regarding the base VoTT
2) Parameter uncertainty arising from estimation of the meta model – this yields additional uncertainty regarding the up rating due to uncertainty in the income elasticity.
3) Uncertainty regarding the values of future income levels (only an issue when forecasting into the future)
4) The weighting procedure adopted for the base VoTT utilises NTS data. There will be some uncertainty reflecting the extent to which the NTS sample is truly reflective of the population.

We have not incorporated uncertainty from source 3 or 4 in the analysis.

We have computed statistical confidence intervals (CIs) for VoTT estimates which incorporate sources 1 and 2 uncertainty identified above. The task is non-trivial for the following reasons:

1) The VoTT from the logit model is a non-linear function of parameters
2) The base VoTT is an average of many individual VoTT estimates. This adds to the computational complexity and excludes using off the shelf software such as LIMDEP to undertake the computations.
3) The VoTT up rating utilises an elasticity estimate from a separate model. This needs incorporating into the computation.

We describe the method in two parts. First we describe the computation of the standard error [SE] of the base VoTT i.e. the weighted average of VoTT estimates from the AHCG-based models. Second we describe the approach in combining this estimate of uncertainty with that arising from up rating by income.

### Part 1: Computing the SE for the Base VoTT

As input into this work, we have obtained the covariance matrix from the models fitted by ITS/Bates to the original AHCG data (two models one for commuting, the other for other non-work trips). This allows us to compute a standard error for the base VoTT.

The models are given as:

$$\text{VTTS} = [\beta_\tau/\beta_C].\left(\frac{Inc}{Inc_0}\right)^{\eta_{Inc}}.\left(\frac{C}{C_0}\right)^{\eta_C}, \qquad (1)$$

where Inc is household income in £'000 pa and C is journey cost in pence (both in 1994 prices), and $Inc_0$ and $C_0$ take the fixed values 35 and 100 respectively. The estimated values for the parameters $\beta_\tau$, $\beta_c$, $\eta_{Inc}$, $\eta_c$ are given in the table below (with t-ratios in brackets), separately for the Commuting and "Other" purposes:

Table 1 Model Parameter estimates

| | **Commute** | **Other** |
|---|---|---|
| $\Delta\tau$ $\beta_\tau$ | -0.10098 (-15.07) | -0.08292 (-19.33) |
| $\Delta c$ $\beta_c$ | -0.02473 (-14.84) | -0.02227 (-18.51) |
| Income Elasticity $\eta_{Inc}$ | 0.35878 (7.58) | 0.15681 (5.49) |
| Cost Elasticity $\eta_C$ | 0.42130 (9.08) | 0.31473 (11.86) |

In order to compute an average VoTT it was necessary to convert cost into distance since NTS data (1995-2000) was chosen to weight the estimates and this was stratified by income and distance. For this purpose, a relationship was derived (Mackie et al, 2003) which suggested that the cost per mile had the average value of 13.2 pence (1994 prices), and that for most of the data a linear assumption between distance and (reported) cost was justified. On this basis, C in the VoT formula can be interpreted as distance (in miles) and the value of $C_0$ can be converted to $D_0$ = 100/13.2 = 7.58 miles (12.2 Km).

It should be noted that the AHCG data was in 1994 prices and 1994 income levels. Thus the formula must be amended to include inflation from 1994 to the desired year (1997 in Mackie et al, 2003, although the spreadsheet for this study up rates to 2002 prices)[2]. This is a separate adjustment to adjusting VoTTs to reflect income changes (see part 2).

Finally VoTT estimates are computed for several distance and income combinations in NTS and weighted by trip distance (as opposed to trips). Thus the overall formula for computing the base VoTT estimate is:

$$\overline{V}\text{ ( Inc, D) } = \frac{\sum_{y \in Inc} \sum_{d \in D} V_{yd} . N_{yd} . D_d}{\sum_{y \in Inc} \sum_{d \in D} N_{yd} . D_d} \qquad (2)$$

[2] The estimate emerging from the [R] code is in 1997 prices (K=1.1 in that computation)

where $N_{yd}$ is the number of NTS journeys for the specified purpose (by all mechanised modes) with distance in distance band d made by persons in household income band y, $D_d$ is the average distance for distance band d in NTS and

$$V_{yd} = K\,[\beta_\tau/\beta_C].\ \left(\frac{Inc'}{Inc'_0}\right)^{\eta_{Inc}}.\left(\frac{D_d}{D_0}\right)^{\eta_C}$$

where *Inc′* represents the NTS household income in £'000 pa, $D_0$ is set to 7.58 miles, *Inc*'$_0$ is set to the value 35*K, and K is a correction for inflation.

In terms of methods to compute the standard error, given that we do not have the original estimation data available for this work, we exclude using bootstrapping resampling (this would require more resource in any case). As such we confine ourselves to methods which utilise the parameter estimates and covariance matrix. The two candidate methods are:

- The Delta method – a first order Taylor series expansion of the variance of a function which has the attractive property of converging to the maximum likelihood variance covariance matrix as the sample size increases
- Simulation approach attributed to Krinsky and Robb (1986[3]) (K&R) – this utilises the result that in the limit, model parameter estimates are distributed normally. We sample for each parameter from a multi-variate normal distribution (utilising the estimated covariance matrix) and compute the base VoTT function. We do this many times (say 10^6 times) and order the set of function evaluations to form an empirical distribution for the function, from which we can compute a variance to form the confidence interval [CI].

However as noted in point 2 above, the base VoTT is an average of several estimates of VoTT for different distances and incomes. This means that the actual base VoTT is the weighted summation of a number of non-linear functions. While conceptually this is not really a complication, in practice most automated routines in off-the-shelf statistical packages will fail to consider such a long function e.g. LIMDEP. As such we have undertaken the computation using bespoke code in the matrix programming language [R]. Further, although it requires a relative large amount of computing time, the

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[3] Krinsky, I., Robb, A., (1986). 'On approximating the statistical properties of elasticities'. Review of Economics and Statistics 68 (4), 715-719.

K&R method is easy to programme and so we proposed to undertake this approach.

The two methods generally give similar results e.g. see Krinsky and Robb (1991)[4] and Greene (2011)[5]. This is because they both rely on the model parameter estimates being normally distributed in large samples. Importantly both are asymptotic approximations to the true parameter distribution.

The [R] code is reproduced in Appendix A. We describe the inputs and outputs of the code in Section 3. The [R] software is freeware and thus is available for download and use by interest parties should it be necessary[6].

## Part 2: Computing the CI for the up rated VoTT estimate

Once we have the standard error for the base VoTT we combine this with the standard error from the income elasticity from the Meta analysis to form a confidence interval for the up-rated VoTT. We compute the standard error using the delta method to combine the uncertainty in the base VoTT with uncertainty in the income elasticity estimate. Thus we use the K&R method to compute the SE of the base VoTT and then use the Delta method to combine this with the uncertainty associated with the income elasticity estimate. We can use the Delta method in this case since the expression for the overall standard error is now relatively simple (unlike when computing the base VoTT). The advantage of using the Delta method for this part of the computation is that it can be automated within the spreadsheet, thus allowing the user greater functionality vis-à-vis having to undertake further K&R runs in [R].

Formally the expression for the point estimate of the up rated VoTT (V*) is given as:

$$V^* = GDP^{\varepsilon_{GDP}}.\bar{V}$$

Where $\bar{V}$ is the base VoTT as defined in (2), $\varepsilon_{GDP}$ is the GDP elasticity, estimated to be 0.8 from the meta-analysis reported in Mackie et al (2003) and GDP is the ratio of GDP of the level of interest to the level in 1994. Thus if GDP was 20% greater than 1994 levels for the forecast year then GDP=1.2.

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[4] Krinsky, I., Robb, A., 1991. On approximating the statistical properties of elasticities: a correction. Review of Economics and Statistics 72 (1), 189{190.
5 Greene, W. H. (2011), Econometric Analysis, 8th Edition, Prentice Hall, New York.
[6] http://www.r-project.org/

The standard error for the estimate of $\varepsilon_{GDP}$ is given in Mackie et al (2003) as 0.1646[7] and the standard error for $\overline{V}$ is given from the K&R simulations in part 1. GDP is assumed to be non-stochastic i.e. we do not take into account uncertainty arising from forecasting GDP. Thus V* comprises two estimated parameters to which we apply the Delta method. Further because the two parameters can be viewed as being based on samples of data which are independent of each other (the meta-analysis sample is independent of the sample for the AHCG sample), then there will be no correlation between the two parameter estimates.[8] This simplifies the computation.

The Delta method proceeds by computing the (asymptotic) variance of V* as:

$$asy.\operatorname{var}(V^*) = \left(\frac{\partial V^*}{\partial \overline{V}}\right)^2 \operatorname{var}(\overline{V}) + \left(\frac{\partial V^*}{\partial \varepsilon_{GDP}}\right)^2 \operatorname{var}(\varepsilon_{GDP})$$
$$asy.\operatorname{var}(V^*) = \left(GDP^{\varepsilon_{GDP}}\right)^2 \left(SE(\overline{V})\right)^2 + \left(\overline{V} \cdot \ln(GDP) \cdot GDP^{\varepsilon_{GDP}}\right)^2 \left(SE(\varepsilon_{GDP})\right)^2 \quad (4)$$

The estimated SE of V* is thus:

$$SE(V^*) = \sqrt{asy.\operatorname{var}(V^*)} \quad (5)$$

The 95% CI is then given as:

$$95\%\ \mathrm{CI}\ \mathrm{V}^* = \left[V^* - 1.96SE(V^*),\ V^* + 1.96SE(V^*)\right] \quad (6)$$

[7] To be more precise, Mackie et al (2003) reports a CI for the meta analysis income elasticity on page 52. From this we can derive the standard error of 0.1646=0.823*(0.4)/2. Note that the point estimate of 0.8 is used in the computations even though the actual point estimate from the meta analysis (IVT model) was 0.823.

[8] Note that in the proposal for this work we said that we may consider correlation between the parameters. When we reflected further on the problem it became evident that this would be misleading.

# 3. EXPLANATION OF THE CODE FOR [R]

Appendix A contains a copy of the code for [R] which will compute the standard error for the base VoTT ($SE(\overline{V})$ in the notation of section 2).

The inputs required for the code are:

- Variance Covariance matrix for the model parameters. Note this is a VC matrix not a correlation matrix. This was sourced from the (unpublished) output from the Mackie et al (2003) analysis of the AHCG data and reproduced in Appendix B for both the Commuting and Other models

- Column vector of parameter estimates (reproduced in Table 1 above)

- Column vector of incomes to be evaluated in VoTT calculation (reproduced in Appendix B)

- Row vector of distances to be evaluated in VoTT calculation (reproduced in Appendix B)

- Weighting matrix for averaging in Part 1. This is the NTS split of trips by distance and income combination (Reproduced in Appendix B)

The key output of the code is the scalar value ‘se.vott’ which is the estimated standard error of $\overline{V}$. In addition, part 2 of the code provides a matrix of standard errors each element being a standard error estimate for the $V_{yd}$ i.e. the individual estimates of VoTT comprising the weighted average. While not directly relevant to the computation of CIs for V*, it does potentially allow the user to consider computing CIs for specific distance and income combinations without the need to rerun [R].

It is not the intention of this section to explain the technical details of the procedure; the code is annotated for this purpose (see Appendix A). The code needs to be run separately for each trip purpose (commuting or other) and also if different weighting matrices are to be used (e.g. to consider the impact of trip distributions for different modes, or more recent NTS data etc).

# 4. DESCRIPTION OF SPREADSHEET

The key output of the spreadsheet is the numerical bounds of the estimated CIs for different GDP levels and associated graphical plots. Given the nature of the modelling process (some being undertaken in [R] others within the spreadsheet) it is important to understand the user functionality in the spreadsheet.

The inputs to the spreadsheet are:

From [R]:

- VoTT SE – se.vott from [R] code – one for ‘commuting’ and one for ‘other’
- Matrix of SE for VoTTs computed for different income/distance combinations – This is not actually used to produce the CIs but is provided for reference in case CIs for specific income/distance combinations are required – there are two matrices from [R], one for ‘commuting’ and one for ‘other’

From Mackie et al (2003)

- GDP elasticity – 0.8
- Standard error for the GDP elasticity – 0.1646

From harris_JB_vot.xls (spreadsheet associated with Mackie et al (2003)):

- VoTT – point estimate of the average VoTT weighted by distance.
- Matrix of computed estimates of VoTTs computed for different income/distance combinations – For use with the matrix of SEs

From National Statistics (or other source)

- GDP per head series. Alternatively the user may change this series for sensitivity testing purposes

In addition the user can set the desired confidence level (e.g. 95%, 99% etc) that is desired and the sheet will automatically adjust the computed CIs. The user can also select the price base for the results to be displayed in. The price level is currently set to 2002 levels.

The work sheets which have the outputs within are ‘Commuting All Modes’ and ‘Other All Modes’ – herein called ‘main sheets’. The following is a list of possible functionality that the user may wish to utilise which just requires changes to the spread sheet and does not require rerunning the [R] code:

- Change the desired confidence level. Amend cell C8 in the main sheets.
- Change the GDP forecasts. Amend cells in row 17 in the main sheets.
- Change the GDP elasticity and/or SE for the GDP elasticity. Amend cells C6 and C7 respectively in the main sheets.
- Consider a different base VoTT e.g. for a specific income and distance combination. Replace the C4 and C5 with the VoTT estimate and simulated SE from the matrices in cells C62:N82 and C87:N107 respectively in the main sheets

The following additional functionality is possible if the [R] code is rerun:

- Change the base model parameters and Variance Covariance matrix of the underlying choice model (e.g. following a re-estimation of the model). This requires rerunning the [R] code with a new vector of parameter estimates and new VC matrix. Then the [R] output can be pasted into a new copy of the main sheet in the workbook, specifically cells C5 and C87:N107. The point estimate for the VoTT (equation (2)) and associated matrix of point estimates for individual distance/income combinations, also needs to be pasted in (this can be requested from the [R] code too).
- Change the weighting matrix. This may be desired for at least three reasons. First more recent NTS data may be available. Second it may be desired to weight by trips rather than distance. Third, VoTTs for individual travel modes may be desired. In this case each model will have a different weighting matrix associated with it. To do this re run the [R] code with a different weighting matrix and paste in the results as described above.

# 5. CONCLUSIONS

## Discussion of results

In the spreadsheet provided with this report, we have produced confidence intervals for the two travel purposes – commuting and other – and for all modes i.e. the weighting matrix used is trip distance by income/distance class for all modes.

Figure 1 and 2 present graphical plots for the 95% confidence intervals for VoTT estimates for commuting and other respectively. As can be seen the key driver of their width is the extent of divergence between GDP per head in the evaluated year and GDP per head in 1994. Thus attaining a precise estimate of the GDP elasticity is very important. Of course this analysis does not include uncertainty due to forecasting GDP; clearly if this was taken into account future years would potentially have CI's substantially wider than shown with a greater width increase for the later years due to compounding of uncertainty.

Figure 1 95% CI for commuting trip purpose VoTT

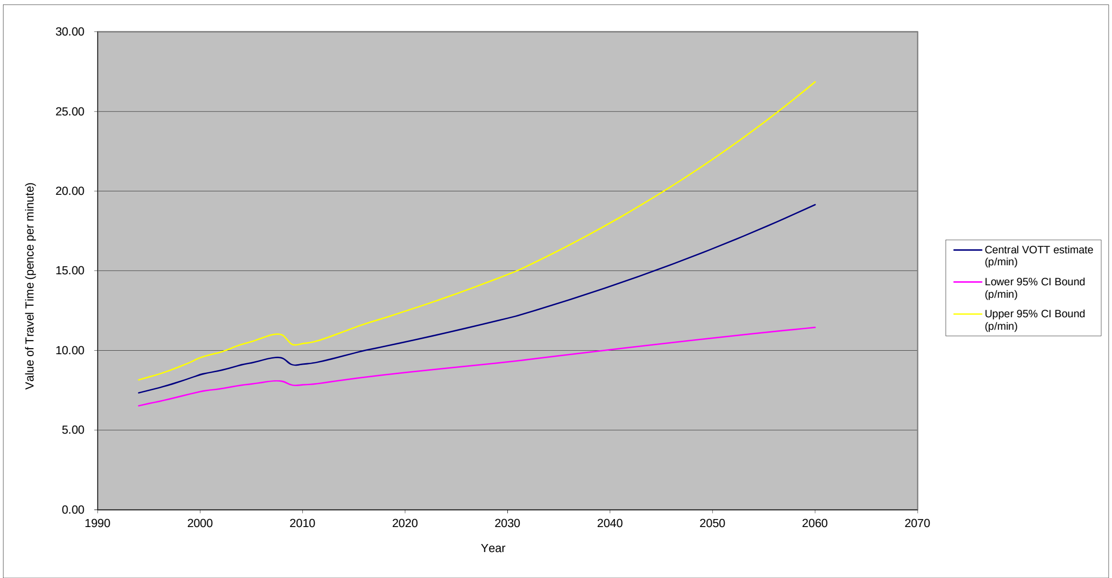
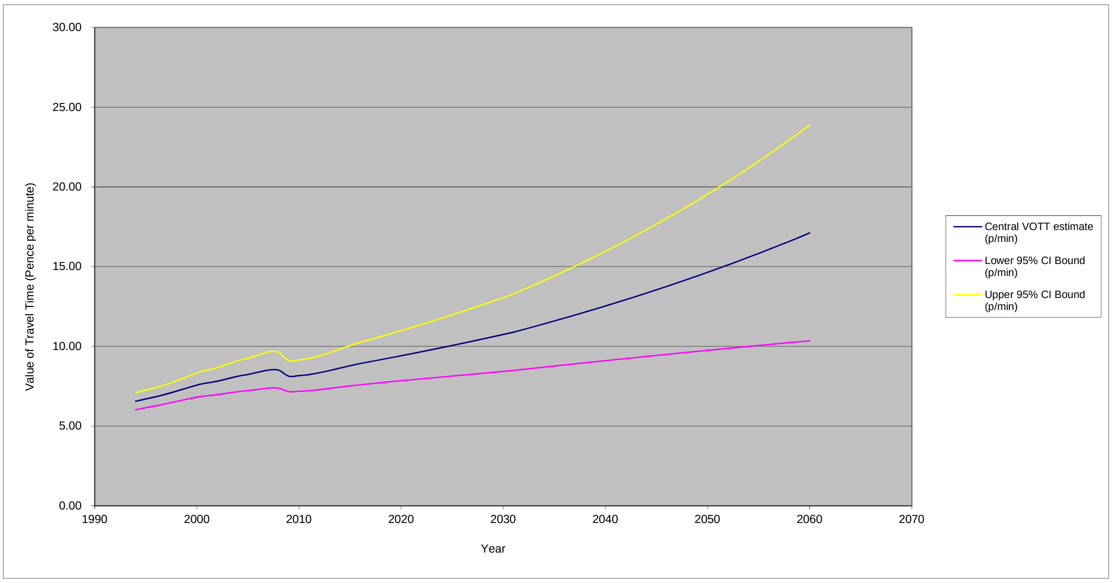


Figure 2 95% CI for other trip purpose VoTT




## Possible ways to amend the analysis following re-sampling and model re-estimation

Should the DfT decide to re-sample and re-estimate its model for VoTT then, as long as the structure stays the same (a logit-based model for base year VoTT and then up rating by a time series based GDP elasticity), the approach can easily be amended to accommodate this. Section 4 describes how to rerun the [R] code and amend the spreadsheet for changes to the ACHG and GDP elasticity.

Clearly a re-sampling (updating) exercise would reduce reliance on the GDP elasticity in the future year CIs. However it should be further emphasised the importance of a precise (and more generally robust) estimate of the GDP elasticity is essential for yielding narrow CIs particularly as the future year increases as uncertainty from this estimated is compounded over time.

The above all assumes that trends in VoTT are explained by the parameters in the model and there is no series break from the past caused for example by changes in trend of improved in-car environment and driver comfort. It would be prudent to check from time to time whether that was the case.

# APPENDIX A – [R] CODE FOR K&R SIMULATIONS

```
# Standard errors for Part 1) for an average VoTT over various
# distance and income levels and for Part 2) for various VoTTs
# evaluated at different distance and income levels

# Phill Wheat
# Institute for Transport Studies
# Univeristy of Leeds
# Code originally produced 16/01/2012
# For latest revision date see file name

#Description:
#This code generates standard errors for averages of different
#distance and income combinations of a travel time function.
#The second part of the code also computes standard errors for
#the individual travel time functions i.e. evaluated at a single
#distance and income combination.
#
#[R] requirements:
#This code uses the base package and in addition requires the installation
of the following packages
#(all available via the R project website):
# - mvtnorm - for the function rmvnorm - the author acknowledges Friedrich
Leisch and
# Fabian Scheipl for the development of this function
#
#Technical details:
#The code utilises the distribution sampling
#approach attributed to Krinsky and Robb (1986). The approach utilises
#that in large samples the ML parameters are distributed multivariate
normal.
#We sample from this distribution and each time construct the statistic of
#interest namely either the average VoTT (part 1) or the individual VoTTs
# (part 2). By default the number of draws is set at 1,000,000 for part 1
# and 100,00 for part 2, however these can be changed by entering a
# different value for drawsp1 and drawsp2 below.

drawsp1<-1000000
drawsp2<-10000

#The input data is:
#new.data1 - VC matrix for parameters. Note this is a VC matrix not a
correlation matrix
#new.data2 - column vector of parameter estimates
#new.data3 - column vector of incomes to be evaluated in VoTT calaculation
#new.data4 - row vector of distances to be evaluated in VoTT calculation
#new.data5 - weighting matrix for averaging in Part 1. Should have same no
#rows as new.data3 and same no columns as new.data4

#The command below is useful for reading in data. Remove the "#" from the
start of
#the line before running the command.

#new.data2 <- read.delim("clipboard",header=FALSE)

# Prelim transformations of input data

vc<-as.matrix(new.data1)
# Following line guarantees VC matrix is positive definite
vc<-0.5*vc+0.5*t(vc)
para<-as.matrix(new.data2)
colnames(vc) <- NULL
rownames(vc)<-NULL
incvec<-as.matrix(new.data3)
rownames(disvec)<-NULL
```

```
disvec<-as.matrix(new.data4)
incmat<-matrix(data=incvec,nrow=nrow(incvec),ncol=ncol(disvec))
dismat<-matrix(data=disvec,nrow=nrow(incvec),ncol=ncol(disvec), byrow=TRUE)
weighting<-as.matrix(new.data5)
colnames(weighting)<-NULL
rownames(weighting)<-NULL


# Part 1 - Create Standard error (se.vott) for the average of VoTs evaluated
for
# all distance and income combinations. The output (se.vott) is a scalar
value


draws1<-as.matrix(rmvnorm(drawsp1,para,vc))

vott<-as.matrix(seq(0,0,length=drawsp1))
vottmat<-1.1*(para[1]/para[3])*((incmat/38.5)^-para[5])*((dismat/7.58)^-
para[6])
vottav<-mean(vottmat)


limitp1<-drawsp1+1
i=1
while(i<limitp1){
parat<-as.matrix(draws1[i,])
vottmatt<-1.1*(parat[1]/parat[3])*((incmat/38.5)^(-
parat[5]))*((dismat/7.58)^-parat[6])
#line below removed as calculates unweighted mean replaced with subsequent
lines
#vott[i]<-mean(vottmatt);
vott[i]<-sum(vottmatt*weighting)/sum(weighting)
i=i+1}

mean.vott<-mean(vott)
se.vott<-sd(vott)



# Part 2 - Create standard errors (se.vott.ind) for each of the VoTs, each
evaluated at a
# different distance and income combinations. The output (se.vott.ind) is a
matrix

draws2<-as.matrix(rmvnorm(drawsp2,para,vc))
#vectorise (sp?) the distance and income matrices
disvec.full<-as.vector(dismat)
incvec.full<-as.vector(incmat)

limitp2<-drawsp2+1
limitp2b<-length(incvec.full)
limitp2a<-length(incvec.full)+1
vott2<-as.matrix(seq(0,0,length=drawsp2))
mean.vec<-as.matrix(seq(0,0,length=limitp2b))
se.vec<-as.matrix(seq(0,0,length=limitp2b))

j=1
while (j<limitp2a) {
i=1
while(i<limitp2){
parat<-as.matrix(draws2[i,])
votttemp<-1.1*(parat[1]/parat[3])*((incvec.full[j]/38.5)^(-
parat[5]))*((disvec.full[j]/7.58)^-parat[6])
vott2[i]<-votttemp;
i=i+1}
```

```
mean.vec[j]<-mean(vott2)
se.vec[j]<-sd(vott2);
j=j+1}

mean.vott.mat<-matrix(data=mean.vec,nrow=nrow(incvec),ncol=ncol(disvec))
se.vott.mat<-matrix(data=se.vec,nrow=nrow(incvec),ncol=ncol(disvec))

#output to text file
write.table(se.vott.mat,"se_vott_mat_other_010212.txt")
```

# APPENDIX B – SUMMARY OF DATA SOURCES FOR [R] CODE

New.data1 – VC matrix – source: unpublished model output

| | | | | | |
|---|---|---|---|---|---|
| 0.00004489000 | -0.00019282131 | 0.00000366758 | 0.00000033835 | -0.00003137409 | -0.00006994800 |
| -0.00019282131 | 0.12623809000 | 0.00031287718 | -0.00037679565 | -0.00137806658 | 0.00433579696 |
| 0.00000366758 | 0.00031287718 | 0.00000289000 | -0.00000678215 | -0.00003417425 | 0.00004590816 |
| 0.00000033835 | -0.00037679565 | -0.00000678215 | 0.00255025000 | -0.00000955460 | 0.00000468640 |
| -0.00003137409 | -0.00137806658 | -0.00003417425 | -0.00000955460 | 0.00223729000 | -0.00026775584 |
| -0.00006994800 | 0.00433579696 | 0.00004590816 | 0.00000468640 | -0.00026775584 | 0.00215296000 |

Other

| | | | | | |
|---|---|---|---|---|---|
| 0.00001849000 | -0.00073333920 | 0.00000057276 | -0.00001638816 | -0.00000343140 | -0.00004011040 |
| -0.00073333920 | 1.13614281000 | 0.00029418840 | 0.00499331514 | -0.00097210080 | 0.00254217150 |
| 0.00000057276 | 0.00029418840 | 0.00000144000 | -0.00000328716 | -0.00001785240 | 0.00002203740 |
| -0.00001638816 | 0.00499331514 | -0.00000328716 | 0.00157609000 | -0.00000339435 | 0.00002314510 |
| -0.00000343140 | -0.00097210080 | -0.00001785240 | -0.00000339435 | 0.00081225000 | -0.00014576325 |
| -0.00004011040 | 0.00254217150 | 0.00002203740 | 0.00002314510 | -0.00014576325 | 0.00070225000 |

New.data2 – parameter vector - source: unpublished model output (also reported in a slightly different way in harris_JB_vot.xls)

Commuting:

| |
|---|
| -0.101 |
| 2.1402 |
| -0.0247 |
| 0.8924 |
| -0.3588 |
| -0.4213 |

Other

| |
|---|
| -0.0829 |
| 7.0671 |
| -0.0223 |
| 0.9554 |
| -0.1568 |
| -0.3147 |

## New.data3 – income classification vector – source: harris_JB_vot.xls

0.85
1.5
2.5
3.5
4.5
5.5
6.5
7.5
8.5
9.5
11.25
13.75
16.25
18.75
22.5
27.5
32.5
37.5
45
62.5
93.75

## New.data4 – distance classification vector – source: harris_JB_vot.xls

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.4 | 1.4 | 2.4 | 3.6 | 7 | 12 | 19 | 29 | 41 | 70 | 140 | 240 |

## New.data5 – Distance weighting matrices – source: harris_JB_vot.xls

### Commuting

| | | 1 Under 1 mile | 2 1 to under 2 miles | 3 2 to under 3 miles | 4 3 to under 5 miles | 5 5 to under 10 miles | 6 10 to under 15 miles | 7 15 to under 25 miles | 8 25 to under 35 miles | 9 35 to under 50 miles | 10 50 to under 100 miles | 11 100 to under 200 miles | 12 200 miles and over |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | |
| 1 Less than £1000 | | 0 | 54 6 | 136 8 | 194.4 | 448 | 288 | 494 | 58 | 0 | 70 | 0 | 0 |
| 2 £1000- £1999 | | 27.2 | 128 8 | 314.4 | 910.8 | 2345 | 1608 | 1577 | 377 | 697 | 630 | 560 | 480 |
| 3 £2000- £2999 | | 11.2 | 95.2 | 218.4 | 363.6 | 2023 | 972 | 2033 | 87 | 0 | 140 | 0 | 480 |
| 4 £3000- £3999 | | 2 | 106.4 | 208 8 | 349 2 | 875 | 1260 | 703 | 348 | 0 | 840 | 0 | 0 |
| 5 £4000- £4999 | | 20.8 | 92.4 | 230.4 | 363 6 | 791 | 804 | 760 | 58 | 0 | 140 | 420 | 0 |
| 6 £5000- £5999 | | 6.4 | 186.2 | 302.4 | 705 6 | 994 | 780 | 266 | 464 | 615 | 0 | 140 | 480 |
| 7 £6000- £6999 | | 22.8 | 222.6 | 420 | 867 6 | 1841 | 540 | 779 | 261 | 82 | 0 | 280 | 0 |
| 8 £7000- £7999 | | 10.8 | 252 | 484 8 | 716.4 | 1617 | 708 | 1083 | 522 | 615 | 0 | 0 | 0 |
| 9 £8000- £8999 | | 20.4 | 271 6 | 388 8 | 982.8 | 2240 | 1728 | 1235 | 174 | 0 | 280 | 280 | 0 |
| 10 £9000- £9999 | | 32 | 319.2 | 513 6 | 1594.8 | 3640 | 1056 | 1767 | 522 | 615 | 980 | 0 | 0 |
| 11 £10000- £12499 | | 56 | 550.2 | 1159 2 | 3603.6 | 8267 | 3504 | 5510 | 986 | 1066 | 840 | 0 | 0 |
| 12 £12500- £14999 | | 90 | 1219.4 | 1948 8 | 5054.4 | 10108 | 7596 | 6954 | 2697 | 2583 | 1750 | 280 | 0 |
| 13 £15000- £17499 | | 138.8 | 1264.2 | 1891 2 | 6076.8 | 12670 | 7692 | 9063 | 4118 | 2173 | 2870 | 700 | 1200 |
| 14 £17500- £19999 | | 92.8 | 1563.8 | 2642.4 | 5814 | 13755 | 9492 | 12027 | 4176 | 3854 | 4480 | 1820 | 1680 |
| 15 £20000- £24999 | | 220 | 2599.8 | 3849 6 | 10393.2 | 28364 | 23976 | 27113 | 10904 | 9143 | 9310 | 2520 | 480 |
| 16 £25000- £29999 | | 146 | 2151.8 | 4248 | 10112.4 | 23317 | 22596 | 30324 | 12325 | 13366 | 17570 | 4760 | 3600 |
| 17 £30000- £34999 | | 127.6 | 1576.4 | 3338.4 | 7862.4 | 21728 | 17868 | 21964 | 13775 | 14965 | 11270 | 2800 | 720 |
| 18 £35000- £39999 | | 148 | 1079.4 | 2155 2 | 5950.8 | 18641 | 16260 | 21128 | 14587 | 13530 | 13930 | 3640 | 1200 |
| 19 £40000- £49999 | | 113.2 | 1257 2 | 2313 6 | 6559.2 | 21336 | 21120 | 28785 | 15747 | 14022 | 20510 | 3360 | 3120 |
| 20 £50000- £74999 | | 83.6 | 873 6 | 1663 2 | 4683.6 | 14203 | 16152 | 24662 | 15341 | 15375 | 11410 | 5180 | 2880 |
| 21 £75000 or more | | 11.6 | 438 2 | 928 8 | 2059.2 | 8988 | 7908 | 11533 | 7366 | 9389 | 9310 | 4620 | 2400 |

## Other

| | | 1 Under 1 mile | 2 1 to under 2 miles | 3 2 to under 3 miles | 4 3 to under 5 miles | 5 5 to under 10 miles | 6 10 to under 15 miles | 7 15 to under 25 miles | 8 25 to under 35 miles | 9 35 to under 50 miles | 10 50 to under 100 miles | 11 100 to under 200 miles | 12 200 miles and over |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | |
| 1 Less than £1000 | | 60.4 | 550 2 | 816 | 1378.8 | 2261 | 2112 | 1957 | 986 | 1927 | 2380 | 2240 | 480 |
| 2 £1000- £1999 | | 141.6 | 1523 2 | 1852 8 | 4471.2 | 8022 | 4056 | 6650 | 3016 | 3485 | 8260 | 7420 | 3600 |
| 3 £2000- £2999 | | 128 | 1048 6 | 1814.4 | 3456 | 6832 | 3972 | 5130 | 2378 | 2788 | 6160 | 6720 | 2880 |
| 4 £3000- £3999 | | 260 | 2580.2 | 4113 6 | 7124.4 | 11172 | 6348 | 6973 | 4785 | 5330 | 6790 | 5880 | 5280 |
| 5 £4000- £4999 | | 402.4 | 3347.4 | 4852 8 | 8211.6 | 13069 | 6684 | 9671 | 3480 | 2624 | 5600 | 7280 | 2640 |
| 6 £5000- £5999 | | 512.4 | 4145.4 | 5817 6 | 10828.8 | 17584 | 10788 | 14573 | 5336 | 7503 | 11480 | 6860 | 4080 |
| 7 £6000- £6999 | | 580 | 4391.8 | 5906.4 | 10371.6 | 18613 | 10584 | 12673 | 7917 | 6273 | 9100 | 7280 | 2640 |
| 8 £7000- £7999 | | 394.4 | 3661 | 5114.4 | 10202.4 | 15736 | 9888 | 9880 | 5452 | 3772 | 8820 | 5880 | 6000 |
| 9 £8000- £8999 | | 369.2 | 3087 | 4804 8 | 8262 | 15498 | 9240 | 9823 | 4147 | 4059 | 7910 | 7280 | 2160 |
| 10 £9000- £9999 | | 363.2 | 3547 6 | 5184 | 9813.6 | 20321 | 10668 | 13433 | 6757 | 6396 | 13370 | 8120 | 4800 |
| 11 £10000- £12499 | | 714 | 6763.4 | 8546.4 | 17319.6 | 34594 | 20724 | 23028 | 11977 | 10414 | 22750 | 15260 | 6960 |
| 12 £12500- £14999 | | 977.6 | 7784 | 10492 8 | 21110.4 | 39760 | 26088 | 30837 | 13949 | 17097 | 24150 | 17220 | 6720 |
| 13 £15000- £17499 | | 870.8 | 7761.6 | 10543 2 | 19623 6 | 41286 | 24852 | 28842 | 17168 | 16646 | 29120 | 22960 | 10320 |
| 14 £17500- £19999 | | 1149.2 | 8713.6 | 12110.4 | 23616 | 42259 | 26604 | 33478 | 19169 | 16933 | 29960 | 31360 | 15600 |
| 15 £20000- £24999 | | 1767.2 | 14551.6 | 20512 8 | 38098 8 | 78134 | 50328 | 60933 | 32886 | 32595 | 70980 | 60760 | 34080 |
| 16 £25000- £29999 | | 1627.2 | 14071.4 | 20380 8 | 36932.4 | 74921 | 46032 | 56031 | 32422 | 32390 | 59010 | 53620 | 26400 |
| 17 £30000- £34999 | | 1325.6 | 11267.2 | 15597 6 | 31075 2 | 58492 | 40284 | 49058 | 29464 | 29192 | 56770 | 52640 | 26400 |
| 18 £35000- £39999 | | 1058.8 | 9531.2 | 11925 6 | 24955 2 | 48993 | 34800 | 41857 | 24853 | 29315 | 46760 | 53200 | 36720 |
| 19 £40000- £49999 | | 1269.2 | 9662.8 | 13336 8 | 27702 | 58674 | 40152 | 52231 | 29290 | 35014 | 65240 | 64960 | 35280 |
| 20 £50000- £74999 | | 846 | 7827.4 | 10994.4 | 22510.8 | 46788 | 32760 | 42218 | 24215 | 30012 | 52430 | 53480 | 26160 |
| 21 £75000 or more | | 467.2 | 4767 | 5913 6 | 12672 | 26026 | 18624 | 24529 | 13543 | 15908 | 37240 | 32340 | 15840 |